

Geometry • Unit 8 Worksheet

Conditional Probability

Name: _____

Date: _____

Conditional probability is $P(A \text{ given } B) = P(A \text{ and } B) / P(B)$. Independence means $P(A \text{ and } B) = P(A) \cdot P(B)$. Tree diagrams, Venn diagrams, and two-way tables help you see what's going on — use them.

Part A • Sample Spaces and Basic Probability

1. A spinner has 8 equal sectors: 3 red, 3 blue, 2 yellow. A six-sided die is also rolled. The experiment is one spin and one roll.

How many outcomes are in the sample space? _____

$P(\text{red and } 6) =$ _____

$P(\text{yellow and even}) =$ _____

Sketch a TREE diagram for the spinner outcomes (red, blue, yellow), then the die outcomes (1-6).

2. From a standard 52-card deck, one card is drawn.

a. $P(\text{heart}) =$ _____

b. $P(\text{face card}) =$ _____ (J, Q, K of any suit)

c. $P(\text{heart AND face card}) =$ _____

d. Using the Addition Rule: $P(\text{heart OR face card}) = P(\text{heart}) + P(\text{face card}) - P(\text{heart AND face card}) =$ _____

Part B • Two-Way Tables and Conditional Probability

3. A survey of 200 students asked about preferred subject and grade level.

	Math	English	Science	Total
9th	30	20	25	75
10th	25	30	20	75

11th-12th	15	20	15	50
Total	70	70	60	200

$P(\text{prefers Math}) = \underline{\hspace{2cm}}$

$P(\text{in 9th grade}) = \underline{\hspace{2cm}}$

$P(\text{prefers Math AND in 9th grade}) = \underline{\hspace{2cm}}$

$P(\text{prefers Math GIVEN 9th grade}) = P(\text{Math AND 9th}) / P(9\text{th}) = \underline{\hspace{2cm}}$

$P(\text{in 9th grade GIVEN prefers Math}) = \underline{\hspace{2cm}}$

Part C • Independent Events

4. Two events A and B are INDEPENDENT if and only if $P(A \text{ and } B) = P(A) \cdot P(B)$, or equivalently, $P(A | B) = P(A)$.

In Problem 3, is 'prefers Math' independent of 'in 9th grade'? Compute both and compare:

What would have to be true of the table for these to be perfectly independent?

5. A coin is flipped 3 times.

Are the flips independent? Yes / No. Why?

$P(\text{all 3 heads}) = \underline{\hspace{2cm}}$

$P(\text{at least 1 head}) = 1 - P(\text{no heads}) = 1 - \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

Part D • Multiplication Rule and Conditional

6. The Multiplication Rule says: $P(A \text{ and } B) = P(A | B) \cdot P(B)$, OR equivalently $P(B | A) \cdot P(A)$.

A bag has 5 red and 7 blue marbles. Two are drawn WITHOUT replacement.

a. $P(\text{first is red}) = \underline{\hspace{2cm}}$

b. $P(\text{second is red} | \text{first is red}) = \underline{\hspace{2cm}}$

c. $P(\text{both red}) = (a) \cdot (b) = \underline{\hspace{2cm}}$

Why does the conditional probability matter here? Why aren't the draws independent?

Part E • Venn Diagrams

7. In a class of 30 students, 18 play sports, 14 play music, and 8 play both.

Sketch a Venn diagram with circles for 'plays sports' and 'plays music'.

How many play sports but NOT music? _____

How many play NEITHER? _____

$P(\text{plays music} \mid \text{plays sports}) =$ _____

Part F • Error Analysis

8. Andre says: "I rolled a die three times and got three 6s. The next roll has a LOWER probability of being a 6, because four 6s in a row is really unlikely." Is Andre correct?

Yes / No:

What's the actual probability of a 6 on the next roll? Why?

What's the name of this fallacy?

9. Mai writes: " $P(\text{rain} \mid \text{umbrella})$ means: the probability of rain, given that someone has an umbrella." Diego writes: " $P(\text{umbrella} \mid \text{rain})$ means: the probability of rain, given an umbrella." Who is correct?

Correct: _____

Explain the notation $P(A \mid B)$ precisely:

Part G • Synthesis / Modeling

10. A test for a disease has 95 percent sensitivity ($P(\text{positive} \mid \text{disease}) = 0.95$) and 90 percent specificity ($P(\text{negative} \mid \text{no disease}) = 0.90$). In the population, 2 percent of people have the disease.

If a random person tests positive, what's the probability they actually have the disease? (Use a 1000-person table.)

Why might this probability be SURPRISINGLY low?

11. OPEN: Design your own probability experiment. State a question, identify the sample space, decide whether the events you're asking about are independent or not, and compute the probability you're after.

Experiment:

Sample space and events:

Computation:
