

Geometry • Unit 7 Worksheet

Circles

Name: _____

Date: _____

Circles are everywhere in geometry. This unit: inscribed angles are half their central angles; tangent lines are perpendicular to the radius at the point of tangency; arc length and sector area scale linearly with the central angle. Radian = arc length / radius.

Part A • Vocabulary and Central/Inscribed Angles

1. Define each in your own words. Sketch a small picture if it helps.

a. Chord:

b. Central angle:

c. Inscribed angle:

d. Arc (minor vs major):

2. An inscribed angle has measure 35 degrees. What is the measure of the central angle that subtends the same arc?

Central angle = _____ degrees

What's the measure of the arc? _____ degrees

State the Inscribed Angle Theorem in one sentence.

Part B • Tangent Lines

3. Line l is tangent to circle O at point P . What is the measure of angle OPl (the angle between the radius OP and the tangent line l)?

Angle = _____ degrees

Why is this always true? Explain the theorem.

4. From an external point E , two tangent lines are drawn to circle O , touching at points P and Q . Show that $EP = EQ$. (Hint: use congruent triangles OEP and OEQ .)

What are the right angles in your figure? Why?

Which triangle congruence theorem applies?

Conclude $EP = EQ$. Explain.

Part C • Cyclic Quadrilaterals

5. Quadrilateral $ABCD$ is inscribed in a circle. Angle $A = 80$ degrees, angle $B = 95$ degrees. Find angle C and angle D . (Hint: opposite angles in a cyclic quadrilateral are supplementary.)

Why are opposite angles supplementary? (Hint: each inscribed angle is half the arc opposite it. The two opposite arcs together make a full circle.)

Part D • Circumscribed and Inscribed Circles

6. The CIRCUMCENTER of a triangle is the intersection of the three perpendicular bisectors of its sides. It is equidistant from the three vertices and is the center of the CIRCUMSCRIBED circle.

Why does the circumcenter exist? Why do the three perpendicular bisectors meet in one point?

Describe how to construct the circumscribed circle of a triangle.

7. The INCENTER of a triangle is the intersection of its three angle bisectors. It is equidistant from the three SIDES and is the center of the INSCRIBED circle.

How is the incenter different from the circumcenter?

Part E • Arc Length, Sector Area, Radians

8. A circle has radius 12. A central angle of 60 degrees subtends a sector.

Arc length: _____ units (formula: $\text{arc} = (\text{theta}/360) \cdot 2\pi r$)

Sector area: _____ square units (formula: $\text{area} = (\text{theta}/360) \cdot \pi r^2$)

Now in radians: 60 degrees = $\pi/3$ radians. What's the arc length using $\text{arc} = r \cdot \text{theta_radians}$?

Why does the radian formula work? (Hint: by DEFINITION, $\theta_{\text{radians}} = \text{arc length} / \text{radius}$.)

Part F • Error Analysis

9. Tyler claims: "Every chord of a circle is a diameter." Is Tyler correct?

Yes / No:

Define both terms precisely:

10. Lin computes: "A circle has radius 10. The arc length for a central angle of 90 degrees is just $(1/4)$ of the circumference $= (1/4)(20\pi) = 5\pi$." Then she says: "In radians, 90 degrees $= \pi/2$. So arc length $= r \cdot \theta = 10 \cdot (\pi/2) = 5\pi$. The two answers agree!"

Verify both computations. Are they the same?

Explain why the two formulas always give the same answer.

11. OPEN: A car tire has a diameter of 26 inches. When the car drives forward 1 mile (5280 feet), through what total angle has each tire rotated? Give your answer in REVOLUTIONS, in DEGREES, and in RADIANS.

Circumference of tire:

Revolutions:

Degrees:

Radians:
