

Geometry • Unit 6 Worksheet

Coordinate Geometry

Name: _____

Date: _____

In this unit, we use coordinates and algebra to prove geometric facts. Pythagorean Theorem gives distances; slopes tell us parallel and perpendicular relationships. Be careful: writing things in coordinate form lets you compute, but you still need a clear argument.

Part A • Coordinate Transformations

1. Triangle ABC has vertices A(1, 2), B(5, 2), C(3, 6). Apply each transformation and give the image vertices.

a. Translation $(x, y) \rightarrow (x + 4, y - 3)$: A'(__ , __) B'(__ , __) C'(__ , __)

b. Reflection $(x, y) \rightarrow (-x, y)$: A'(__ , __) B'(__ , __) C'(__ , __)

c. Dilation $(x, y) \rightarrow (2x, 2y)$: A'(__ , __) B'(__ , __) C'(__ , __)

Which of (a), (b), (c) preserve distance? Which produce a SIMILAR (but not congruent) image?

Part B • Distance and Circles

2. Find the distance between each pair of points using the distance formula. Show the Pythagorean Theorem set-up.

a. (2, 3) and (7, 15): Distance = _____

b. (-4, 1) and (8, 6): Distance = _____

3. Write an equation for each circle.

a. Center (3, -5), radius 7: _____

b. Center origin, passes through (6, 8): _____

c. What is the center and radius of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$?

Part C • Slopes and Equations of Lines

4. Line L passes through (2, 5) and (7, -3).

Slope of L: $m =$ _____

Equation of L in point-slope form: _____

Equation of the line PARALLEL to L through (1, 1): _____

Equation of the line PERPENDICULAR to L through (1, 1): _____

5. Slopes of parallel lines are EQUAL; slopes of perpendicular lines are NEGATIVE RECIPROCALs (product = -1).

Explain WHY parallel lines have equal slopes. (Hint: think about translations.)

Explain WHY perpendicular slopes multiply to -1. (Hint: think about a 90 degree rotation about the origin: $(x,y) \rightarrow (-y, x)$.)

Part D • Coordinate Proof

6. Show that the quadrilateral with vertices A(0, 0), B(5, 0), C(8, 4), D(3, 4) is a PARALLELOGRAM. (Strategy: show both pairs of opposite sides are parallel by computing slopes.)

Points: A(0, 0), B(5, 0), C(8, 4), D(3, 4)

Slope of AB: _____ Slope of DC: _____

Slope of AD: _____ Slope of BC: _____

Conclusion (which sides are parallel, and why this makes ABCD a parallelogram):

7. Show that the triangle with vertices P(-1, 2), Q(4, 4), R(2, 9) is a RIGHT TRIANGLE.

Find the slope of each side:

Which two sides are perpendicular? Why?

Alternative: verify with the converse of the Pythagorean Theorem using distances.

Part E • Partitioning Segments

8. Point P partitions segment AB such that $AP:PB = 1:3$. $A = (2, 4)$, $B = (10, 12)$.

Find P. (Use weighted averages.) $P = (\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$

What if instead $AP:PB = 3:1$? $P = (\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$

The midpoint of AB corresponds to which ratio? _____

Part F • Error Analysis and Synthesis

9. Mai writes: "The line $y = 3x + 2$ is perpendicular to the line $y = -3x + 5$ because the slopes are opposites." Is Mai correct?

Yes / No:

What's the correct condition for perpendicularity? What is the perpendicular slope to 3?

10. Diego writes: " $x^2 + y^2 + 6y = 0$ isn't a circle because there's no constant term and no x term." Is Diego right? Rewrite this equation in center-radius form.

Yes / No:

Complete the square:

Center: _____ Radius: _____

11. OPEN: Prove that the diagonals of a rectangle are equal in length using COORDINATES. Place a rectangle conveniently on the coordinate plane (use variables for the dimensions).

Place the rectangle: vertices at $(0, 0)$, $(a, 0)$, (a, b) , $(0, b)$.

Find the lengths of the two diagonals using the distance formula:

Conclude:
