Geometry • Unit 5 Worksheet

Solid Geometry

Name: _____ Date: _____

When you scale a solid by factor k: lengths x k, surface area x k squared, volume x k cubed. This pattern is the key idea of the unit. Cavalieri's Principle says solids with the same height and matching cross-section areas have the same volume.

Part A • Solids of Rotation and Cross-Sections

1. Describe the 3D solid that results when each 2D shape is rotated about the given axis.

a. A rectangle, rotated about one of its sides. Solid: _____

b. A right triangle, rotated about a leg. Solid: _____

c. A semicircle, rotated about its diameter. Solid: _____

2. A cube of side length 6 is sliced by a plane.

a. Describe the cross-section when the plane is parallel to a face.

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b. Describe the cross-section when the plane is perpendicular to a body-diagonal and passes through the center. (Shape and dimensions if you can.)

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c. Is it possible to slice the cube to get a PENTAGONAL cross-section? Try to describe one.

Part B • Scaling: lengths, areas, volumes

3. A rectangular prism (box) has dimensions 4 x 6 x 10. It is scaled by factor 3.

New dimensions: _____ x _____ x _____

Surface area of original: _____ sq units. Of scaled: _____ sq units. Ratio: _____

Volume of original: _____ cubic units. Of scaled: _____ cubic units. Ratio: _____

Explain in your own words WHY surface area scales by k squared and volume by k cubed.

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4. A toy company makes two similar bottles. The smaller holds 250 mL; the larger holds 2000 mL. The smaller bottle is 12 cm tall.

Scale factor (use volumes):

Height of larger bottle:

If the label on the smaller bottle has area 30 cm squared, area of larger label:

Part C · Volume Formulas

5. Compute the volume of each.

a. Cylinder: radius 5 cm, height 12 cm. V = _____ cm cubed (use pi or leave in terms of pi)

b. Cone: same dimensions as (a). V = _____ cm cubed

c. Sphere: radius 5 cm. V = _____ cm cubed (formula: V = (4/3)·pi·r cubed)

What is the ratio of the cone's volume to the cylinder's volume? Why is it 1/3?

6. An OBLIQUE cylinder has a circular base of radius 4 and the slanted height is 15, but the perpendicular height from base to top is only 12. What's the volume?

Which height do we use in the cylinder volume formula? Why? (Hint: Cavalieri's Principle.)

V = _____

Part D · Cavalieri's Principle

7. State Cavalieri's Principle in your own words.

Statement:

Example application: a right cylinder and an oblique cylinder both have circular base of area A and the same perpendicular height h. Why do they have the same volume?

Part E · Density and Modeling

8. A spherical iron ball has radius 6 cm. Iron has density 7.87 g/cm cubed. What's the ball's mass?

Volume:

Mass = volume x density. Mass = _____ g

9. MODELING: A factory makes ice cream cones (cones) with radius 3 cm and height 12 cm, topped with a hemispherical scoop of ice cream with the same radius. If the ice cream melts and fills only the cone (no overflow), will the cone hold all the melted ice cream?

Volume of hemisphere of ice cream:

Volume of cone:

Does it fit? By how much room is left over (or how much overflows)?

Part F • Error Analysis and Open Modeling

10. Andre says: "If you double all the dimensions of a fish tank, you need twice as much water to fill it." Is Andre correct?

Yes / No:

If no, what's the correct factor? Justify.

11. OPEN: Two cones have the same volume. Cone A has radius 4 and some height h_A. Cone B has radius 2 and some height h_B. What is the relationship between h_A and h_B?

Set up equations:

Relationship between heights:

Intuition check: which cone is taller? Why?