

Geometry • Unit 3 Worksheet

Similarity

Name: _____

Date: _____

Congruence is a special case of similarity (scale factor = 1). Similar triangles have proportional sides and equal angles, NOT equal sides. Don't conflate the two.

Part A • Dilations

1. Dilate point $P(4, 6)$ about the origin with scale factor k . Give the image.

a. $k = 2$: $P' = (\quad , \quad)$

b. $k = 1/2$: $P' = (\quad , \quad)$

c. $k = -1$: $P' = (\quad , \quad)$

Which of (a), (b), (c) preserves distances? Why?

2. A triangle with vertices $A(0,0)$, $B(4,0)$, $C(0,3)$ is dilated about the origin by scale factor 3. Find the image vertices A' , B' , C' .

Compute the perimeters of the original and image. What's the ratio?

Compute the areas of the original and image. What's the ratio?

Notice: lengths scale by k , areas scale by k squared. Why?

Part B • Similar vs Congruent

3. For each, state whether the figures must be SIMILAR, must be CONGRUENT, both, or neither.

- a. Two squares. _____
- b. Two equilateral triangles. _____
- c. Two rectangles. _____
- d. Two regular hexagons with the same side length. _____
- e. Two right triangles where both have a 30 degree angle. _____

4. Triangles ABC and DEF have angle A = angle D = 50 degrees and angle B = angle E = 70 degrees. Are they similar? Are they congruent?

Similar? Yes / No. Why?

Congruent? Yes / No. Why?

What theorem about similarity did you use?

Part C • Using Similar Triangles

5. Triangles ABC and DEF are similar with the correspondence A to D, B to E, C to F. You know $AB = 6$, $BC = 8$, $AC = 10$, $DE = 9$.

What is the scale factor from ABC to DEF? _____

Find DF and EF. Show your work.

If angle A = 36 degrees, what is angle D? Why?

6. INDIRECT MEASUREMENT: At the same time of day, a 6-ft tall person casts a 4-ft shadow. A nearby flagpole casts a 22-ft shadow. How tall is the flagpole?

Hint: the two triangles formed (person + shadow, pole + shadow) are similar because the sun's rays are parallel and both objects are vertical.

Set up a proportion:

Solve. The flagpole is _____ feet tall.

Why does the assumption 'sun's rays are parallel' make the triangles similar? (Be specific.)

Part D • Pythagorean Theorem via Similarity

7. In right triangle ABC with right angle at C, drop an altitude from C to side AB, meeting AB at D. This creates THREE similar triangles: ABC, ACD, CBD.

Why are these three triangles all similar? Identify the equal angles.

Using the proportion $AB/AC = AC/AD$ (from triangle ABC similar to ACD), what does AC squared equal?

Similarly, what does BC squared equal?

Add AC squared + BC squared. What do you get? Explain why this proves the Pythagorean Theorem.

Part E • Error Analysis

8. Han claims: "If two figures are similar, then their areas have the same ratio as their sides." Is Han correct?

Yes / No:

Justify with a specific example (e.g., two squares with side lengths 2 and 6).

9. Mai writes: "Triangle ABC has sides 6, 8, 10. Triangle DEF has sides 9, 12, 15. They have proportional sides but different angles, so they are NOT similar." Critique.

What is correct in Mai's statement?

What is INCORRECT?

What is the actual relationship between these two triangles?

Part F • Synthesis

10. A photograph is enlarged by scale factor 2.5. The original photograph is 4 inches by 6 inches and uses 12 square inches of ink to print.

Find the new dimensions of the enlarged photograph.

How much ink does the enlarged photo use? (Assume ink coverage is proportional to area.)

If you also enlarge the volume of a 3D photo frame around it by 2.5, by what factor does the volume scale?

11. OPEN: A line through the midpoints of two sides of a triangle is parallel to the third side and half as long (the 'midsegment theorem'). Sketch why this is true using similarity.

Diagram and explanation:
