Algebra 2 · Unit 5 Worksheet

Exponential Functions and Equations

Name: ____ Date: ____________________

This unit deepens exponential thinking: rational inputs, growth/decay over different intervals, the introduction of the number e, and logarithms as a way to solve for an unknown exponent.

Part A · Growth and Decay

1. A bacteria culture starts at 200 cells and triples every hour.

a. Write a function P(t) for the number of cells after t hours.

b. How many cells after 5 hours? _____________ After 8 hours? _____________

c. How many cells half an hour after the start? Show your reasoning — does the function still work for $t = 0.5$?

Part B · Changing the Time Unit

2. An investment grows by 8% per YEAR. Find the equivalent growth rate per MONTH.

a. Write the yearly model $V(t) = V_0 \cdot (\_\_\_\_)^t$ for $V_0 = 1000$.

b. If the monthly multiplier is m, then $m^{12} = 1.08$. Solve for m.

c. Is the monthly rate the same as $8/12 = 0.667\%$? Why or why not?

Part C · Logarithms

3. Evaluate without a calculator.

a. $\log_2(32)$ = _____ $\log_2(1)$ = _____ $\log_2(1/8)$ = _____

b. $\log_{10}(1000)$ = _____ $\log_{10}(0.01)$ = _____

c. $\log_3(81)$ = _____ $\log_5(\sqrt{5})$ = _____

d. In your own words: log_b(x) is the number that answers what question?

Part D · Solving Exponential Equations

4. Solve each equation. Use logarithms when needed.

a. 2^x = 64 x = _______

b. 3^x = 20 x ≈ _______ (show how you used a log)

---

c. 5 · 2^t = 100 t = _______

---

d. 10^(2x+1) = 500 x = _______

Part E · The Number e

5. Consider compound interest on $1: (1 + 1/n)^n as n grows.

a. Fill in approximate values (use a calculator).

b. What number does (1 + 1/n)^n approach? Name it. _______

c. Why is e considered the 'natural' base for continuous growth?

---

n	(1 + 1/n)^n
1	2
2	2.25
10	≈ 2.59374
100	
1000	
1,000,000	

Part F · Modeling with Exponential and Logarithmic

6. A radioactive sample has a half-life of 12 years. You start with 100 grams.

a. Write A(t) for amount remaining after t years.

b. How long until 25 grams remain? Use reasoning.

c. How long until 10 grams remain? Use logs.

Part G · Two Methods

7. Solve 4^x = 7 using TWO different log bases.

Method 1: log base 10	Method 2: log base e (ln)

Do you get the same numerical answer? Why must you?

Part H · Error Analysis

8. Diego wrote: "log(a · b) = log(a) · log(b)." Is he correct?

☐ Yes ☐ No — Test with a = 10, b = 100. Compare log(1000) with log(10)·log(100).

What IS the correct log rule for log(a · b)?

---

9. Lin claims: "Since exponential functions grow forever, they MUST eventually be larger than any polynomial." Is this true?

Part I · Reflection

10. Why are logarithms useful? Give a real-world reason (acidity, decibels, earthquakes, etc.) where logarithms make a difference manageable.