Algebra 2 · Unit 4 Worksheet

Complex Numbers and Rational Exponents

Name: _____ Date: _____

This unit extends exponents to fractions, solves equations with squares and roots, and introduces imaginary and complex numbers. Then you use them to solve every quadratic equation.

Part A · Rational Exponents

1. Rewrite each in radical form, then evaluate exactly.

a. 8^(1/3) = _____ = _____

b. 16^(3/4) = _____ = _____

c. 25^(−1/2) = _____ = _____

d. Explain why a^(1/n) means the nth root of a.

Part B · Squares and Square Roots

2. Compare these two equations carefully.

a. Solve $x^2 = 49$. _____

b. Solve $\sqrt{x} = 7$. _____

c. Why does (a) have TWO solutions but (b) has only ONE?

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3. Solve $\sqrt{(x + 5)} = x - 1$.

a. Show your algebra (square both sides).

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b. Check each candidate solution back in the ORIGINAL equation.

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c. Which solution is extraneous? Why does squaring sometimes introduce extra answers?

Part C · Introducing i

4. We define i so that $i^2 = -1$.

a. Compute i^3, i^4, i^5, i^6 as simple powers of i or as integers.

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b. Why is there a need for i? (What equation could not be solved without it?)

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c. Compute i^{50}. (Hint: divide 50 by 4 and look at the remainder.)

Part D · Arithmetic with Complex Numbers

5. Let $z = 3 + 2i$ and $w = 1 - 4i$.

a. $z + w =$ _____

b. $z - w =$ _____

c. $z \cdot w =$ _____ (show your distribution)

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d. If you are asked for z^2, what is it? Show work.

Part E · Quadratic Equations with Complex Solutions

6. Solve $x^2 + 4x + 13 = 0$ using TWO methods.

Method 1: Complete the square	Method 2: Quadratic formula

Both methods should give the same complex solutions. What are they?

Part F · The Discriminant

7. For each equation, use the discriminant $b^2 - 4ac$ to predict the number and type of solutions (without solving).

a. $x^2 - 6x + 9 = 0$ Discriminant = _____ Solutions: ☐ 2 real ☐ 1 real ☐ 2 complex

b. $2x^2 + 3x - 5 = 0$ Discriminant = _____ Solutions: ☐ 2 real ☐ 1 real ☐ 2 complex

c. $x^2 + 2x + 5 = 0$ Discriminant = _____ Solutions: ☐ 2 real ☐ 1 real ☐ 2 complex

d. Explain WHY the sign of the discriminant determines real vs. complex.

Part G · Error Analysis

8. Han wrote: "√(−9) = −3 because −3 · −3 = 9."

☐ Correct ☐ Incorrect — Explain.

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9. Mai squared both sides of √x = −4 and got x = 16. She is excited because she 'solved it.' Is she right?

Check by substituting x = 16 back into √x = −4. What goes wrong?

How many ACTUAL solutions does √x = −4 have? Explain.

Part H · Connecting It All

10. Show that (a + bi)(a − bi) is ALWAYS a real number. What is it?

a. Expand and simplify.

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b. Why is this useful when DIVIDING complex numbers?
