

Algebra 2 • Unit 3 Worksheet

Rational Functions and Equations

Name: _____

Date: _____

Rational functions are ratios of polynomials. You will reason about asymptotes, solve rational equations (watching for extraneous solutions), and use polynomial identities — including the formula for a finite geometric sum.

Part A • Domain and Asymptotes

1. Let $f(x) = (x + 3) / (x - 2)$.

a. What is the domain of f ? Why is $x = 2$ excluded?

b. Find the vertical asymptote. _____

c. Find the horizontal asymptote. Explain how the degrees of numerator and denominator determine it.

d. What is the x -intercept? The y -intercept?

Part B • End Behavior of Rational Functions

2. For each rational function, determine the horizontal asymptote (or describe end behavior).

a. $g(x) = 4x^2 / (2x^2 + 7)$. HA: $y = \underline{\hspace{2cm}}$. Why?

b. $h(x) = 5x / (x^2 + 1)$. HA: $y = \underline{\hspace{2cm}}$. Why?

c. $k(x) = (x^3 + 1) / (x - 4)$. Does this have a horizontal asymptote? Why or why not?

Part C • Hole vs. Asymptote

3. Let $r(x) = (x^2 - 4) / (x - 2)$.

- Simplify $r(x)$. _____
- Is $x = 2$ a vertical asymptote or a HOLE in the graph? Explain.

c. What is $r(3)$? What value would the function 'want' to have at $x = 2$?

Part D • Solving Rational Equations + Extraneous Solutions

4. Solve: $1 / (x - 3) = 5 / (x + 1)$.

- Show your work clearly.

b. Check your solution(s) by substituting back into the ORIGINAL equation.

c. Why is checking important here, even more than usual?

5. Solve: $x / (x - 2) = 2 / (x - 2) + 1$.

a. Show your algebra.

b. Is your solution EXTRANEIOUS? If so, what does the equation actually have for solutions?

Part E • Application — Average Cost

6. A company has \$5000 fixed costs and pays \$20 to produce each item. Let $A(n)$ be the average cost per item when n items are made.

a. Write $A(n)$ as a rational function.

b. Find $A(100)$ and $A(10,000)$.

c. What happens to $A(n)$ as $n \rightarrow \infty$? Why does this make sense in the real-world context?

d. Will $A(n)$ ever ACTUALLY reach \$20? Why or why not?

Part F • Polynomial Identity

7. Prove that $(x - 1)(x^2 + x + 1) = x^3 - 1$ is an identity.

a. Expand the left side step by step.

b. Does it equal the right side for ALL values of x , or only some?

c. Use the identity to factor $27 - 1$ (with $x = 3$). What value do you get?

Part G • Sum of a Finite Geometric Sequence

8. Compute $S = 1 + 3 + 9 + 27 + 81 + 243 + 729$ using the formula for the sum of a finite geometric sequence: $S = a(1 - r^n) / (1 - r)$.

a. Identify a , r , and n .

b. Plug in and simplify.

c. Verify by adding directly.

Part H • Error Analysis

9. Lin solves $3/x = 2$ by cross-multiplying: " $3 \cdot 1 = 2 \cdot x$, so $x = 3/2$." Is she right?

☐ Yes ☐ No — Explain what went wrong if anything.

10. Why does multiplying both sides of an equation by an expression containing x sometimes INTRODUCE extraneous solutions? Explain in your own words.
