Algebra 1 · Unit 5 Worksheet

Functions

Name: _____ Date: _____

Part A · Function Notation and Definition

1. For each rule, decide if it defines y as a FUNCTION of x. Justify.

a. $y = 3x - 7$. Function Not

b. $x = y^2$ (y is the dependent variable). Function Not

c. The graph passes the vertical-line test, intersecting the line $x = 4$ at exactly one point. Function Not

d. A table where $x = 2$ appears twice, with $y = 5$ and $y = 5$. Function Not

e. A table where $x = 2$ appears twice, with $y = 5$ and $y = 7$. Function Not

2. Given $f(x) = 2x^2 - 3x + 1$, find:

a. $f(0)$ = _____

b. $f(-2)$ = _____

c. $f(a + 1)$ = _____ (expand and simplify)

d. Solve $f(x) = 1$ for x. _____

Part B · Reading Graphs of Functions

3. A graph of g(t) shows the temperature of a cup of coffee over 30 minutes. The graph starts at (0, 90°C), drops quickly, and levels off near 22°C at t = 30.

a. Estimate g(0). What does it mean in context?

b. Is g(t) increasing or decreasing? What does that tell you about the coffee?

c. What is happening near t = 30? What's the room temperature, approximately?

d. Is the function linear? Why or why not?

Part C · Average Rate of Change

4. Given the function f with values f(1) = 4, f(3) = 16, f(7) = 64.

a. Compute the average rate of change of f from x = 1 to x = 3. _____

b. Compute the average rate of change of f from x = 3 to x = 7. _____

c. Are the rates equal? What does that suggest about f?

d. If f WERE linear, what would have to be true about these two rates?

Part D · Domain, Range, and Piecewise

5. A taxi charges a flat $3.50 plus $2.25 per mile. Let C(m) = cost of a ride of m miles.

a. Write a rule for C(m). C(m) = _____

b. What is a reasonable DOMAIN for this function?

c. What is the RANGE corresponding to your domain?

d. If the company decides to charge only WHOLE-DOLLAR amounts (rounding up), would C still be the same function? Why or why not?

6. A piecewise function:

a. Find h(−4), h(0), h(2), h(3), h(5).

b. Is h continuous at x = 0? at x = 3? Show your check.

c. What is the domain? The range?

$$h(x) = \begin{cases} 2x + 1 & \text{if } x < 0 \\ x^2 & \text{if } 0 \le x \le 3 \\ 12 - x & \text{if } x > 3 \end{cases}$$

Part E · Absolute Value and Transformations

7. Compare $f(x) = |x|$ with $g(x) = |x - 3| + 2$.

a. Describe how g's graph relates to f's. (Shifts? Stretches?)

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b. What is the vertex of f? Of g? f-vertex: _____ g-vertex: _____

c. Why does the +2 OUTSIDE the absolute value shift VERTICALLY, while the −3 INSIDE shifts HORIZONTALLY (and to the right, not left)?

Part F · Inverse Functions

8. Consider the function $f(x) = 3x - 6$.

a. Find f-inverse of x.

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b. Show that f(f-inverse(5)) = 5. (Two-step check.)

c. If f converts dollars to euros at a fixed rate, what does f-inverse do? Use this to explain WHY "undoing" defines an inverse.

Part G · Error Analysis and Modeling

9. Han writes $f(x + h) = f(x) + h$. Diego writes $f(x) \cdot g(x) = (f \cdot g)(x)$. Decide whether each is ALWAYS true, SOMETIMES true, or NEVER true. Give specific examples.

a. Han's claim about f(x + h):

b. Diego's claim about products:

10. A function f is shown by a graph that looks like a parabola opening upward with vertex at (2, −1).

a. On what interval is f INCREASING? On what interval DECREASING?

b. What is the minimum value of f? Where does it occur?

c. Why does f have NO maximum value?

11. Sketch a graph of a function that is: positive on $(-\infty, 0)$, negative on $(0, 3)$, positive on $(3, \infty)$, with a maximum at $x = -1$ and a local minimum at $x = 2$.