Divisibility & Remainders

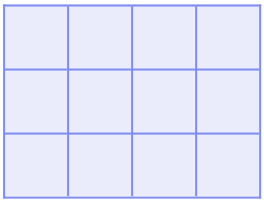
A GUIDE

How to share things fairly, spot leftovers, and read a number's secrets from its digits.

Sharing with nothing left over

Imagine you have 12 cookies and 3 friends. You deal them out one at a time, like cards: one for each friend, then another round, then another. Everyone ends up with exactly 4 cookies, and your hands are empty.

That is what dividing evenly means. 12 ÷ 3 = 4 with nothing left over, so we say 12 is divisible by 3. When a number divides evenly, you can arrange the objects in a perfect rectangle — equal rows, no stragglers.



The remainder: what's left in your hand

Now try 14 cookies with 4 friends. You deal out 3 rounds — that uses 12 cookies — but the 2 left in your hand are not enough for another full round. Those 2 are the remainder: the part that would not share out evenly.

You can see it on a number line. Count by 4s: 0, 4, 8, 12, 16. The number 14 lands between 12 and 16 — it is 2 steps past 12. That gap is the remainder. One rule always holds: the remainder must be smaller than the number you are dividing by, because if it were bigger, you could deal out another full round.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Worked example What is 23 ÷ 5?

1. Count by 5s toward 23: 5, 10, 15, 20 — that is 4 fives.
2. One more five would be 25, which is too big, so stop at 20.
3. Find what is left: 23 − 20 = 3.

Answer: 4 remainder 3

The last-digit trick (2, 5, and 10)

Here is something surprising: to check if a number divides evenly by 2, 5, or 10, you only need to look at its last digit. You can ignore everything else — even in a huge number like 4,981,730.

It works because every number is made of tens plus a last digit, and tens always split evenly by 2, 5, and 10. So only the last digit can cause a leftover. Divisible by 2? Last digit is 0, 2, 4, 6, or 8. Divisible by 5? Last digit is 0 or 5. Divisible by 10? Last digit is 0.

Try counting by 5s on a number line: every stop ends in 0 or 5, forever. That pattern is the whole trick.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Worked example Is 6,340 divisible by 2, by 5, and by 10?

1. Look only at the last digit: it is 0.
2. 0 is even, so 6,340 is divisible by 2.
3. 0 is also on the 0-or-5 list, so it is divisible by 5.
4. A last digit of 0 means it is divisible by 10 too.

Answer: Yes to all three

The digit-sum trick (3 and 9)

For 3 and 9, the last digit is not enough — but there is an even cooler trick. Add up all the digits. If that sum is divisible by 3, so is the whole number. If the sum is divisible by 9, the whole number is too.

Why does adding digits work? A ten is 9 + 1, and a hundred is 99 + 1. The 9s and 99s always divide evenly by 3 and 9, so all that is left to check is the little +1s — one for each digit. That is exactly what the digit sum counts.

If the digit sum is still a big number, just add its digits again. You can repeat until the number is small enough to check in your head.

Worked example Is 741 divisible by 3? Is it divisible by 9?

1. Add the digits: 7 + 4 + 1 = 12.
2. 12 = 3 × 4, so 741 is divisible by 3.
3. But 12 is not divisible by 9 (9 fits once with 3 left over), so 741 is not divisible by 9.

Answer: **Divisible by 3: yes. Divisible by 9: no.**

Remainders in the real world

Remainders show up whenever real things refuse to split perfectly — people, cars, boxes, days of the week. The math gives you a quotient and a remainder, but the real question decides what to do with them.

Sometimes the remainder is the answer: "How many cookies are left for you?" Sometimes you round up: if 26 kids need cars that hold 4, six cars carry 24 kids and 2 kids are still standing on the curb — so you need a 7th car. And sometimes you round down: with 26 eggs, you can only fill 2 full cartons of 12. Same division, three different answers. Always ask what the leftover means.

Worked example A camp has 30 kids. Each tent sleeps 4. How many tents does the camp need?

1. Divide: 30 ÷ 4 = 7 remainder 2, since 7 × 4 = 28.
2. The remainder is 2 kids — real kids who still need somewhere to sleep.
3. Round up: 7 full tents plus 1 more for the last 2 kids.

Answer: **8 tents**

Remember

★ For 2, 5, and 10, only the last digit matters — the rest of the number can't cause a leftover.

★ For 3 and 9, add the digits; if the sum is still big, add its digits again.

★ A remainder is always smaller than the number you divide by — if it isn't, you can deal out another round.

★ In word problems, the leftover is real: decide whether to keep it, round up, or round down before you answer.

Now you try

1) You have 10 stickers and 2 friends. Can you share the stickers equally with nothing left over?
   Answer: _____

2) What is left over when you share 17 marbles equally among 5 kids?
   Answer: _____

3) Is 74 divisible by 2?
   Answer: _____

4) Is 85 divisible by 10?
   Answer: _____

5) Is 522 divisible by 3?
   Answer: _____

6) Is 4,608 divisible by 9?

Answer: _____

7) You pack 50 muffins into boxes of 6. After filling as many full boxes as you can, how many muffins are left over?

Answer: _____

8) What is the remainder when 100 is divided by 9? (Hint: try the digit-sum trick.)

Answer: _____

Answers — check yourself

1. yes
2. remainder 2
3. yes
4. no
5. yes
6. yes
7. 2
8. remainder 1