

Counting Puzzles

A GUIDE

How to count outfits, line-ups, and handshakes without writing out every single one

Three Shirts, Two Pants... How Many Outfits?

Imagine your closet has 3 shirts: red, green, and yellow. It also has 2 pairs of pants: blue and black. How many different outfits can you make?

You could list them all. Red shirt with blue pants. Red shirt with black pants. Then do the same for green, then for yellow. Every shirt gets a turn with every pair of pants.

Count the list and you get 6 outfits. Notice something sneaky: 3 shirts times 2 pants is exactly 6. That is not a coincidence.

Shirt	Pants
red	blue
red	black
green	blue
green	black
yellow	blue
yellow	black

The Multiplying Move

Here is the big idea, and it works everywhere: when you make one choice AND then another separate choice, you multiply the number of options. Each shirt opens up 2 pants choices, so 3 shirts give you $3 \times 2 = 6$ combos.

The best part is that it keeps going. Add a third choice, like 2 hats, and you multiply again: $3 \times 2 \times 2 = 12$ full outfits. Every new choice multiplies what you already had.

This move is called the multiplication principle. It turns a huge listing job into one quick calculation.

3

 $\times 2$
 $\rightarrow$

6

Worked example An ice cream stand has 4 flavors, 3 toppings, and 2 cone types. How many different ice creams can you build if you pick one of each?

1. You make three separate choices: a flavor, a topping, and a cone.
2. Multiply the flavor choices by the topping choices: $4 \times 3 = 12$.
3. Multiply that by the cone choices: $12 \times 2 = 24$.

Answer: 24 different ice creams

Lining Things Up

Now try a different puzzle: how many ways can you arrange 3 toys in a row? This time the choices are not separate closets. They shrink as you go.

For the first spot in the row, you have 3 toys to pick from. Once one is placed, only 2 toys are left for the second spot. Then just 1 toy remains for the last spot.

Shrinking choices still multiply: $3 \times 2 \times 1 = 6$ ways. Every time you line things up in order, count down like this: start at how many things you have, and multiply all the way to 1.

3

 $\times 2$
 $\rightarrow$

6

 $\times 1$
 $\rightarrow$

6

Worked example You have 4 books and one shelf. How many different orders can you put them in?

1. The first spot on the shelf has 4 possible books.
2. The second spot has 3 books left, so multiply: $4 \times 3 = 12$.
3. The third spot has 2 books left: $12 \times 2 = 24$.
4. The last spot has only 1 book left: $24 \times 1 = 24$.

Answer: 24 orders

The Handshake Puzzle

Five friends meet, and everyone shakes hands with everyone else exactly once. How many handshakes happen? Careful, this one has a twist.

Each of the 5 friends shakes 4 other hands, so you might say $5 \times 4 = 20$. But that counts every handshake twice: once as you shaking my hand, and once as me shaking yours. It is the same handshake either way.

So fix the double count by dividing by 2: $20 \div 2 = 10$ handshakes. The rule for any group is: number of people $\times$ one less than that, then divide by 2.

Worked example There are 7 players at a chess club, and every player plays every other player exactly once. How many games is that?

1. Each of the 7 players faces 6 opponents, giving $7 \times 6 = 42$.
2. But every game got counted twice, once from each player's side.
3. Divide by 2 to fix the double count: $42 \div 2 = 21$.

Answer: 21 games

Remember

- ★ If the choices are separate things (a shirt AND pants), multiply the number of options for each.
- ★ Lining things up in order? Count down and multiply: 4 things means $4 \times 3 \times 2 \times 1$.
- ★ Handshakes and high-fives get counted twice (you-to-me and me-to-you), so multiply, then divide by 2.
- ★ Not sure? List every combo for a small case first — if your list matches your multiplication, your rule works.

Now you try

- 1) You have 2 hats and 3 scarves. If an outfit is one hat plus one scarf, how many different outfits can you make?
Answer: _____
- 2) A snack bar offers 4 kinds of juice and 2 kinds of muffin. How many juice-and-muffin combos are there if you pick one of each?
Answer: _____
- 3) A sandwich shop has 3 breads, 4 fillings, and 2 sauces. You pick one of each. How many different sandwiches can you build?
Answer: _____
- 4) You have 3 trophies to place in a row on a shelf. How many different orders can you place them in?
Answer: _____
- 5) Four friends line up single file for a photo. How many different line-ups are possible?
Answer: _____
- 6) Six kids are at a party, and every kid high-fives every other kid exactly once. How many high-fives happen?
Answer: _____

7) You own 5 shirts and some pairs of pants. If you can make exactly 30 different shirt-and-pants outfits, how many pairs of pants do you own?

Answer: _____

8) Eight teams enter a tournament where every team plays every other team exactly once. How many games are played in total?

Answer: _____

Answers — check yourself

1. 6
2. 8
3. 24
4. 6 ways
5. 24 ways
6. 15
7. 6
8. 28