Grade 7 Mathematics

Full Syllabus

Illustrative Mathematics® 6-8 Math (v.360)

9 Units · Unit Goals · Section Goals · Lesson-by-Lesson Outline

Source: https://accessim.org/6-8/grade-7?a=teacher

Course Overview

Grade 7 of the Illustrative Mathematics 6-8 curriculum is organized into 9 units. Students study scale drawings, deepen their work with proportional relationships, measure circles, apply proportions to percentages, do arithmetic with rational numbers, work with expressions/equations/inequalities, study angles, triangles, and prisms, explore probability and sampling, and culminate with an optional Putting It All Together unit.

The unit sequence is:

- Unit 1: Scale Drawings
- Unit 2: Introducing Proportional Relationships
- Unit 3: Measuring Circles
- Unit 4: Proportional Relationships and Percentages
- Unit 5: Rational Number Arithmetic
- Unit 6: Expressions, Equations, and Inequalities
- Unit 7: Angles, Triangles, and Prisms
- Unit 8: Probability and Sampling
- Unit 9: Putting It All Together

Each unit is divided into lettered sections (A, B, C, D as applicable), and each section is a sequence of daily lessons. Optional lessons are labeled “(Optional).”

Unit 1: Scale Drawings

Unit Goal

Students study scaled copies of plane figures and scale drawings of real-world objects, learning that lengths in a scaled copy are products of the original lengths and a scale factor while angles are preserved.

Unit Narrative

In this unit, students study scaled copies of plane figures and scale drawings of real-world objects. Students learn that all lengths in a scaled copy are the result of multiplying the original lengths by a scale factor. Also, the angle measures in a scaled copy are the same as in the original figure. This work builds on what students learned in previous grades about measuring lengths, areas, and angles. This unit provides a geometric context to preview the type of reasoning that students will use with proportional relationships later in grade 7. It also lays the foundation for grade 8 work on dilations and similarity. Students begin the unit by looking at copies of a picture and describing what differentiates scaled and non-scaled copies. They calculate scale factors and draw scaled copies of figures. Note that the study of scaled copies is limited to pairs of figures that have the same orientation—in other words, they are not rotations or reflections of each other. In grade 8, students will extend their knowledge of scaled copies when they study translations, rotations, reflections, and dilations. Next, students study scale drawings. They see that the principles and strategies that they used to reason about scaled copies of figures can also be used with scale drawings. They use scale drawings to calculate actual lengths and areas, and they create scale drawings.

Section A: Scaled Copies

Section Goals

- Determine whether a figure is a scaled copy of another figure, by examining corresponding side lengths and angle measures.
- Draw a scaled copy of a figure using a given scale factor.

Section Narrative

In this section, students study scaled copies of pictures and plane figures. Students begin by looking at copies of a picture, some of which are to scale and some of which are not. They use their own words to describe what differentiates scaled and non-scaled copies of a picture. As the section progresses, students learn that a scaled copy is created when all the lengths in the original figure are multiplied by a consistent scale factor and all the angle measures stay the same. Next, students draw scaled copies. They learn that when the scale factor is greater than 1, the copy will be larger, and when it is less than 1, the copy will be smaller. In the last lesson of the section, students use manipulatives to explore the area of scaled copies. This lesson is optional because the standards do not require students to recognize the pattern that the area changes by the square of the scale factor by which the sides changed.

Lessons

1. **What Are Scaled Copies?** — Let's explore scaled copies.
2. **Corresponding Parts and Scale Factors** — Let's describe features of scaled copies.
3. **Making Scaled Copies** — Let's draw scaled copies.
4. **Scaled Relationships** — Let's find relationships between scaled copies.
5. **The Size of the Scale Factor** — Let's look at the effects of different scale factors.

6. **Scaling and Area (Optional)** — Let's build scaled shapes and investigate their areas.

Section B: Scale Drawings

Section Goals

- Create a scale drawing given the actual measurements of the object or given another scale drawing at a different scale.
- Explain how to use scales and scale drawings to calculate actual distances and areas.

Section Narrative

In this section, students learn about scale drawings and use them to solve problems. They see that the principles and strategies that they used to reason about scaled copies of figures can be used with scale drawings. At first, students work with scales that involve units (for example, “1 cm represents 10 km”). They interpret and draw maps and floor plans. They also reproduce a scale drawing at a different scale. Later, students work with scales that do not include units (for example, “the scale is 1 to 100”). They learn to express scales with units as scales without units, and vice versa. They determine appropriate scales to use, given restrictions in the size of the drawing. They study the relationship between regions and lengths in scale drawings, including calculating the actual area given a scale drawing. This section includes an optional lesson in which students work with maps and solve problems involving constant speed.

Lessons

1. **Scale Drawings** — Let’s explore scale drawings.
2. **Scale Drawings and Maps (Optional)** — Let’s use scale drawings to solve problems.
3. **Creating Scale Drawings** — Let’s create our own scale drawings.
4. **Changing Scales in Scale Drawings** — Let’s explore different scale drawings of the same actual thing.
5. **Scales without Units** — Let’s explore a different way to express scales.
6. **Units in Scale Drawings** — Let's use different scales to describe the same drawing.

Section C: Let’s Put It to Work

Section Narrative

In this final section, students have the opportunity to apply their thinking from throughout the unit. Since this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions. The lesson in this section is optional because it offers additional opportunities to practice standards that are not a focus of the grade.

Lessons

1. **Draw It to Scale (Optional)** — Let’s draw a floor plan.

Unit 2: Introducing Proportional Relationships

Unit Goal

Students develop the idea of a proportional relationship, working with relationships represented in tables, as equations, and on graphs.

Unit Narrative

In this unit, students develop the idea of a proportional relationship. They work with proportional relationships that are represented in tables, as equations, and on graphs. This builds on grade 6 work with equivalent ratios and helps prepare students for the study of linear functions in grade 8. Students begin by looking at tables. In a table of equivalent ratios, a multiplicative relationship between a pair of rows is given by a scale factor, while the multiplicative relationship between the columns is given by a unit rate. Students learn that the relationship between pairs of values in the two columns is called a "proportional relationship," and the unit rate that describes this relationship is called a "constant of proportionality." Next, students use equations to represent proportional relationships and solve problems. They determine whether given tables and equations could represent a proportional relationship. Then students investigate graphs of proportional relationships. They recognize that the graph of a proportional relationship is a straight line through the origin. They interpret points on the graph. By the end of the unit, students should be comfortable working with common contexts associated with proportional relationships (such as constant speed, unit pricing, and measurement conversions) and be able to determine whether or not a relationship is proportional. In a later unit, students will apply proportional reasoning to solve multi-step problems and to calculate more complex rates.

Section A: Representing Proportional Relationships with Tables

Section Goals

- Determine the constant of proportionality for a proportional relationship represented in a table, and interpret it in context.
- Explain how to calculate unknown values in a table that represents a proportional relationship.

Section Narrative

In this section, students work with proportional relationships that are represented in tables. Students begin by reviewing situations that involve ratios and identifying which ratios are equivalent. Next, they learn to view a table of equivalent ratios as representing a proportional relationship. Students learn that all entries in one column of the table can be obtained by multiplying entries in the other column by the same number. This number is called the constant of proportionality. Students use tables to solve problems involving proportional relationships.

Lessons

1. **One of These Things Is Not Like the Others** — Let’s remember what equivalent ratios are.
2. **Introducing Proportional Relationships with Tables** — Let’s solve problems involving proportional relationships using tables.
3. **More about Constant of Proportionality** — Let’s solve more problems involving proportional relationships using tables.

Section B: Representing Proportional Relationships with Equations

Section Goals

- Use an equation to solve problems involving a proportional relationship.
- Write an equation to represent a proportional relationship, given a table or a description of the situation.

Section Narrative

In this section, students use equations to represent proportional relationships and solve problems. They learn that any proportional relationship can be represented by an equation, where the constant is the constant of proportionality. Students begin by revisiting some of the same contexts that they previously examined with tables. They think about how the repeated calculations can be expressed with an equation. Next, students see that there are two different equations that represent each situation, depending on which quantity is regarded as being proportional to the other. The two constants of proportionality in those two equations are reciprocals of each other. Then students write equations and use them to solve problems involving proportional relationships in new contexts.

Lessons

1. **Proportional Relationships and Equations** — Let's write equations describing proportional relationships.
2. **Two Equations for Each Relationship** — Let's investigate equations that represent proportional relationships.
3. **Writing Equations to Represent Relationships** — Let's use equations to solve problems involving proportional relationships.

Section C: Comparing Proportional and Nonproportional Relationships

Section Goals

- Determine whether the values in a table could represent a proportional relationship.
- Use a table to determine whether an equation represents a proportional relationship.

Section Narrative

In this section, students decide whether two quantities are in a proportional relationship. First, students examine tables. They calculate a unit rate for each row of the table and check whether these rates are all the same. If so, the relationship could be proportional. Next, students work with equations. Students use an equation to create a table and determine whether the relationship is proportional. Later, they decide whether a situation involves a proportional relationship based on characteristics of the situation, that is, whether or not it involves a constant rate.

Lessons

1. **Comparing Relationships with Tables** — Let's explore how proportional relationships are different from other relationships.
2. **Comparing Relationships with Equations** — Let's develop methods for deciding if a relationship is proportional.
3. **Solving Problems about Proportional Relationships** — Let's solve problems about proportional relationships.

Section D: Representing Proportional Relationships with Graphs

Section Goals

- Determine whether a given graph represents a proportional relationship.
- Identify the constant of proportionality from the graph of a proportional relationship.
- Interpret points on the graph of a proportional relationship.

Section Narrative

In this section, students work with proportional relationships that are represented with graphs. Students learn that the graph of any proportional relationship lies on a line through the origin that passes through Quadrant I. First, students compare graphs of proportional and nonproportional relationships. Next, they create and interpret graphs of proportional relationships. Later, they compare different proportional relationships that are graphed on the same axes and see that the constant of proportionality indicates the steepness of the line.

Lessons

1. **Introducing Graphs of Proportional Relationships** — Let's see how graphs of proportional relationships differ from graphs of other relationships.
2. **Interpreting Graphs of Proportional Relationships** — Let's read stories from the graphs of proportional relationships.
3. **Using Graphs to Compare Relationships** — Let's graph more than one relationship on the same grid.
4. **Two Graphs for Each Relationship** — Let's use tables, equations, and graphs to answer questions about proportional relationships.

Section E: Let's Put It to Work

Section Narrative

In this final section, students have the opportunity to apply their thinking from throughout the unit. As this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions. The first lesson is optional because it provides an opportunity for additional practice applying the concepts from this unit to model with mathematics.

Lessons

1. **Four Representations (Optional)** — Let's contrast relationships that are and are not proportional in four different ways.
2. **Using Water Efficiently** — Let's investigate saving water.

Unit 3: Measuring Circles

Unit Goal

Students apply their knowledge of proportional relationships to measuring circles, learning the relationships between radius, diameter, circumference, and area.

Unit Narrative

In this unit, students apply their knowledge of proportional relationships to the context of measuring circles. They learn the relationships between radius, diameter, circumference, and area of circles and use these relationships to solve problems. This builds on students' work from previous grades with perimeter and area of polygons. Students will build on this work in grade 8 when they study the volume of spheres, cylinders, and cones. The unit begins with activities designed to help build up students' vocabulary for describing circles more precisely. The terms "center," "radius," "diameter," and "circumference" are introduced. Then students investigate the relationship between circumference and diameter and see that it is a proportional relationship. They apply this relationship to solve problems. Next, students explore the area of circular regions. They see an informal derivation that shows where the area formula comes from and then use this formula to solve problems. Finally, students solve problems that require deciding whether the situation relates to the circumference or area of a circle.

Section A: Circumference of a Circle

Section Goals

- Recognize that there are proportional relationships between the circumference, diameter, and radius of circles and express these relationships using equations.
- Solve problems involving circumference.

Section Narrative

In this section, students apply proportional reasoning to solve problems involving circumference. The section begins with students measuring squares. Students learn to take into account potential measurement error when deciding whether a list of related measurements could represent a proportional relationship. Next, students investigate circles. They learn the terms radius, diameter, and circumference, as well as a more precise meaning of the term circle. Then, they repeat the process they previously used while measuring squares to measure circles. They recognize that the circumference of a circle is proportional to its diameter. Students estimate the constant of proportionality, and then pi is introduced. Students apply the relationship to solve problems. The last lesson of the section focuses on the proportional relationship between the number of times a wheel rotates and the distance it travels. This lesson is optional because it provides more opportunities to practice with proportional relationships that involve circumference.

Lessons

1. **How Well Can You Measure?** — Let's see how accurately we can measure.
2. **Exploring Circles** — Let's explore circles.
3. **Exploring Circumference** — Let's explore the circumference of circles.
4. **Applying Circumference** — Let's use pi to solve problems.
5. **Circumference and Wheels (Optional)** — Let's explore how far different wheels roll.

Section B: Area of a Circle

Section Goals

- Justify that the area of a circle can be calculated with the formula involving pi and the square of the radius.
- Recognize that the area of a circle is not proportional to its diameter or radius.
- Solve problems involving the area of a circle.

Section Narrative

In this section, students learn to calculate the area of a circle and apply this to solve problems. The section begins by reviewing how to decompose regions to calculate their area. Next, students use similar strategies to estimate the areas of various circles. They see that the relationship between the diameter and area of a circle is not proportional. Then, students explore informal derivations of the fact that the area of a circle is equal to pi times the square of its radius. Each derivation involves decomposing the circle and rearranging the pieces. One method results in a parallelogram while the other results in a triangle. Students apply the area formula to solve problems. They learn to express exact answers in terms of pi as well as use approximations of pi to express answers numerically.

Lessons

1. **Estimating Areas** — Let's estimate the areas of weird shapes.
2. **Exploring the Area of a Circle** — Let's investigate the areas of circles.
3. **Relating Area to Circumference** — Let's rearrange circles to calculate their areas.
4. **Applying Area of Circles** — Let's find the areas of shapes made up of circles.

Section C: Let's Put It to Work

Section Narrative

In this final section, students have the opportunity to apply their thinking from throughout the unit. As this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions. The last lesson in this section is optional because it offers additional opportunities to practice standards that are not a focus of the grade.

Lessons

1. **Distinguishing Circumference and Area** — Let's contrast circumference and area.
2. **Stained-Glass Windows (Optional)** — Let's use circumference and area to design stained-glass windows.

Unit 4: Proportional Relationships and Percentages

Unit Goal

Students deepen their understanding of proportional relationships and percentages, solving multi-step problems and working with situations that involve fractional amounts.

Unit Narrative

In this unit, students deepen their understanding of proportional relationships and percentages. They solve multi-step problems and work with situations that involve fractional amounts. This builds on the work students did in grade 6 with ratios, rates, and percentages as well as previous units in grade 7 with proportional relationships. Students will build on this work in high school with exponential functions representing compounded percent increase and decrease. Students begin the unit by revisiting constant rates, but this time the given values are fractional amounts. To determine the unit rate for the situation, students must compute the quotient of two fractions. Students also make sense of situations where an increase or decrease is expressed as a fraction of the initial amount. They create diagrams and apply the distributive property to generate expressions that represent these situations. They also use long division to write fractions as decimals, including their first introduction to repeating decimals. Next, students make sense of situations where an increase or decrease is expressed as a percentage of the initial amount. They continue creating diagrams and writing equations to represent the situations. They solve for any one of the three quantities—the initial amount, the final amount, or the percentage of the change—given the other two quantities. They also reason about fractional percentages. Then students apply percent increase and decrease to solve problems in a variety of real-world situations, such as tax, tip, interest, markup, discount, depreciation, and commission. Lastly, students make sense of situations where the difference between a correct measurement and an incorrect measurement is expressed as a percentage of the correct amount.

Section A: Proportional Relationships with Fractions

Section Goals

- Create algebraic expressions that represent a situation involving adding or subtracting a fraction of the initial value.
- Solve problems about proportional relationships with fractional quantities.
- Use long division to generate a decimal representation of a fraction.

Section Narrative

In this section, students work with proportional relationships that involve fractional values. First they investigate scaled copies where the side lengths include fractions. Then students solve problems involving constant rates between fractional quantities. They use tables to make sense of proportional situations and solve for unknown values. This section also helps prepare students for learning about percent increase and decrease. Early in the section, students review how to calculate percentages. Later in the section, students examine situations where an increase or decrease is expressed as a fraction of the initial amount. They create diagrams and equations to represent these situations. They use the distributive property to generate equivalent expressions and also use long division to convert fractions to decimals.

Lessons

1. **Lots of Flags** — Let's explore the U.S. flag.
2. **Ratios and Rates with Fractions** — Let's calculate some rates with fractions.

3. **Revisiting Proportional Relationships** — Let's use constants of proportionality to solve more problems.
4. **More than That, Less than That** — Let's use fractions to describe increases and decreases.
5. **Say It with Decimals** — Let's use decimals to describe increases and decreases.

Section B: Percent Increase and Decrease

Section Goals

- Create algebraic expressions or equations that represent a situation involving percent increase or decrease.
- Use diagrams to solve problems involving percent increase or decrease.

Section Narrative

In this section, students work with percent increase and decrease. First they interpret verbal descriptions and diagrams. They find the new amount after being given the initial amount and the percentage of the increase or decrease. Then students create their own diagrams and solve problems where the final amount is known but the initial amount or the percentage of the change is unknown. Students relate verbal descriptions and algebraic expressions, which is especially useful for solving for an unknown initial amount. In the last lesson of the section, students reason about percentages that are not whole numbers, which prepares them for more real-world applications of percent increase and decrease.

Lessons

1. **Increasing and Decreasing** — Let's use percentages to describe increases and decreases.
2. **One Hundred Percent** — Let's solve more problems about percent increase and percent decrease.
3. **Percent Increase and Decrease with Equations** — Let's use equations to represent increases and decreases.
4. **Part of a Percent** — Let's explore percentages smaller than 1%.

Section C: Applying Percentages

Section Goals

- Calculate measurement error, and express it as a percentage of the actual value.
- Solve problems involving tax, tip, simple interest, markup, markdown, or commission.

Section Narrative

In this section, students work with real-world examples of percent increase and decrease. Some applications of percent increase include sales tax, tips, interest, and markup. Applications of percent decrease include discounts, depreciation, and commission. However, it is not necessary for students to memorize the meanings of these terms. The goal is for students to be able to write equations and solve problems without needing to draw a diagram first. Then students work with percent error. They interpret situations where the difference between a correct value and an incorrect value is expressed as a percentage of the correct value. They solve for any one of the three quantities—the correct amount, the incorrect amount, or the percentage of the error—given the other two quantities.

Lessons

1. **Tax and Tip** — Let's learn about sales tax and tips.

2. **Percentage Contexts** — Let’s learn about more situations that involve percentages.
3. **Solving Multi-step Percentage Problems** — Let’s solve more problems about sales tax and discounts.
4. **Measurement Error** — Let’s use percentages to describe how accurately we can measure.
5. **Percent Error** — Let’s use percentages to describe other situations that involve error.

Section D: Let’s Put It to Work

Section Narrative

In this final section, students have the opportunity to apply their thinking from throughout the unit. As this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions.

Lessons

1. **Changes on the Earth (Optional)** — Let’s use percentages to make sense of changes on Earth.
2. **Posing Percentage Problems** — Let's explore how percentages are used in the news.

Unit 5: Rational Number Arithmetic

Unit Goal

Students perform operations on rational numbers, which are all numbers that can be written as a positive or negative fraction.

Unit Narrative

In this unit, students perform operations on rational numbers, which are all numbers that can be written as a positive or negative fraction. This builds on grade 6 work with interpreting, comparing, and plotting rational numbers. It prepares students for a later unit when they will solve equations whose coefficients and constants are rational numbers. Students begin by revisiting how signed numbers are used to represent quantities above and below a reference point, such as measurements of temperature and elevation. They use tables and number line diagrams to represent changes in temperature or elevation. They extend addition and subtraction from fractions to all rational numbers. Next, students examine multiplication and division. They work with constant velocity, which is a signed number that indicates direction and speed. This allows products of signed numbers to be interpreted in terms of position, direction of movement, and time before or after a specific point. Students use the relationship between multiplication and division to understand how division extends to rational numbers. Then students work with expressions that use the four operations on rational numbers. They also solve problems that involve interpreting negative numbers in context. They solve linear equations where the coefficients and constants are rational numbers. The focus of these lessons is representing situations with equations and what it means for a number to be a solution for an equation, rather than methods for solving equations. Such methods are the focus of a later unit.

Section A: Adding and Subtracting Rational Numbers

Section Goals

- Apply addition and subtraction of signed numbers to represent situations and solve problems.
- Calculate the sum or difference of two rational numbers.

Section Narrative

In this section, students make sense of adding and subtracting signed numbers. They begin by considering how changes in temperature and elevation can be represented with tables, number line diagrams, expressions, and equations. Then they use these examples to generalize rules for adding signed numbers. They apply the rules to signed numbers that represent money contexts, including deposits, withdrawals, and debts. Next, students use the relationship between addition and subtraction to begin subtracting signed numbers. They interpret subtraction problems as addition with an unknown addend and use a number line to find the answer. From this, they generalize a rule for subtraction. They also learn the convention that “the difference of two numbers” means subtracting the second from the first, and this value can be positive or negative depending on the relative location of the numbers on the number line. Lastly, students apply their understanding of adding and subtracting signed numbers to represent situations and solve problems.

Lessons

1. **Interpreting Negative Numbers** — Let's review what we know about signed numbers.
2. **Changing Temperatures** — Let's add signed numbers.
3. **Changing Elevation** — Let's solve problems about adding signed numbers.

4. **Money and Debts** — Let's apply what we know about signed numbers to money.
5. **Representing Subtraction** — Let's subtract signed numbers.
6. **Finding Differences** — Let's bring addition and subtraction together.
7. **Adding and Subtracting to Solve Problems** — Let's apply what we know about signed numbers to different situations.

Section B: Multiplying and Dividing Rational Numbers

Section Goals

- Apply multiplication and division of signed numbers to represent situations and solve problems.
- Calculate the product or quotient of two rational numbers.

Section Narrative

In this section, students make sense of multiplying and dividing signed numbers. They begin by exploring constant velocity. This builds on their previous work with constant speed but uses signed numbers to represent positions and movement in opposite directions from a reference point. Students calculate the resulting position for a given velocity and time. Then they use these examples to generalize rules for multiplying signed numbers. An optional lesson provides more opportunities for students to build fluency with multiplying rational numbers. Next, students use the relationship between multiplication and division to begin dividing signed numbers. They interpret division problems as multiplication with an unknown factor. From this, they generalize rules for dividing signed numbers. Lastly, students apply their understanding of multiplying and dividing signed numbers to represent situations and solve problems.

Lessons

1. **Multiplying Rational Numbers (Part 1)** — Let's use signed numbers to represent movement.
2. **Multiplying Rational Numbers (Part 2)** — Let's multiply signed numbers.
3. **Multiply! (Optional)** — Let's get more practice multiplying signed numbers.
4. **Dividing Rational Numbers** — Let's divide signed numbers.
5. **Negative Rates** — Let's apply what we know about signed numbers.

Section C: Four Operations with Rational Numbers

Section Goals

- Apply the four operations with rational numbers to solve problems.
- Solve an equation of the form $px + q = r$ or $p(x + q) = r$, where p, q, and r are rational numbers.
- Write an equation to represent a situation involving rational numbers.

Section Narrative

In this section, students solve problems involving the four operations with rational numbers. First, they evaluate algebraic expressions involving signed numbers. They extend their understanding of algebraic notation to include negative numbers, by using "next to" notation (that is, numbers or variables placed next to each other, either with or without parentheses) to represent multiplication and the fraction bar to represent division. Next, students examine new contexts that can be represented with rational numbers. They represent situations with equations, solve equations, and interpret their solutions in context of the situation. The focus of this section is representing situations with equations and what it

means for a number to be a solution to an equation, rather than methods for solving equations. Such methods are the focus of a later unit.

Lessons

1. **Expressions with Rational Numbers** — Let’s develop our signed number sense.
2. **Solving Problems with Rational Numbers** — Let’s use all four operations with signed numbers to solve problems.
3. **Solving Equations with Rational Numbers** — Let’s solve equations that include negative values.
4. **Representing Contexts with Equations** — Let’s write equations that represent situations.

Section D: Let’s Put It to Work

Section Narrative

In this final section, students have the opportunity to apply their thinking from throughout the unit. As this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions.

Lessons

1. **The Stock Market** — Let's learn about the stock market.

Unit 6: Expressions, Equations, and Inequalities

Unit Goal

Students deepen their algebraic reasoning as they write and solve equations and inequalities, and work with more complex equivalent expressions.

Unit Narrative

In this unit, students deepen their algebraic reasoning as they write and solve equations of the forms px + q = r and p(x + q) = r and inequalities of similar forms. Students also work with equivalent expressions that are more complex than what they have seen previously. This builds on grade 6 work with simpler equations and equivalent expressions. Students will build on this work in grade 8 when they solve equations that have a variable on both sides of the equal sign and when they work with systems of equations. Students begin the unit by making sense of situations that involve both multiplication and addition. They represent such situations with tape diagrams and with equations. They see that different diagrams and equations can represent the same situation, and they use diagrams to find solutions to equations. Next, students consider hanger diagrams as another way to represent equations. The diagrams help students understand solving equations in terms of "doing the same thing to each side of the equation." Students examine different pathways for solving the same equation and consider whether one method is more efficient than another. Then students apply what they have learned about equations to inequalities. They write inequalities to represent situations and solve inequalities by reasoning about the related equation. Lastly, students work with equivalent linear expressions that are more complex due to having more terms, more parentheses, and negative rational numbers. Students use properties of operations to justify why the expressions are equivalent.

Section A: Representing Situations of the Form px + q = r and p(x + q) = r

Section Goals

- Create diagrams and equations to represent situations involving multiplication and addition.

Section Narrative

In this section, students make sense of situations involving both multiplication and addition. They represent these situations with tape diagrams and equations. They see that different diagrams and equations can represent the same situation, and they use diagrams to find solutions to equations. Then students write equations to represent situations and solve problems, including more complicated problems involving percent increase and percent decrease.

Lessons

1. **Relationships between Quantities** — Let's try to solve some new kinds of problems.
2. **Reasoning about Contexts with Tape Diagrams** — Let's use tape diagrams to make sense of different kinds of stories.
3. **Reasoning about Equations with Tape Diagrams** — Let's see how equations can describe tape diagrams.
4. **Reasoning about Equations and Tape Diagrams (Part 1)** — Let's see how tape diagrams can help us answer questions about unknown amounts in stories.
5. **Reasoning about Equations and Tape Diagrams (Part 2)** — Let's use tape diagrams to help answer questions about situations where the equation has parentheses.

6. **Distinguishing between Two Types of Situations** — Let's think about equations with and without parentheses and the kinds of situations they describe.

Section B: Solving Equations of the Form px + q = r and p(x + q) = r and Problems That Lead to Those Equations

Section Goals

- Solve equations of these forms, including those that involve fractions, decimals, and negative numbers, and explain the solution method.

Section Narrative

In this section, students use hanger diagrams to make sense of solving equations. The diagrams help students understand solving equations in terms of doing the same thing to each side of the equation. Students examine different pathways for solving the same equation and consider whether one method is more efficient than another. They extend solving equations to include negative numbers. Finally, students apply equations to solve problems, including more complicated problems involving percent increase and percent decrease.

Lessons

1. **Reasoning about Solving Equations (Part 1)** — Let's see how a balanced hanger is like an equation and how moving its weights is like solving the equation.
2. **Reasoning about Solving Equations (Part 2)** — Let's use hangers to understand two different ways of solving equations with parentheses.
3. **Dealing with Negative Numbers** — Let's show that doing the same to each side works for negative numbers too.
4. **Different Options for Solving One Equation** — Let's think about which way is better when we solve equations with parentheses.
5. **Using Equations to Solve Problems** — Let's use tape diagrams, equations, and reasoning to solve problems.
6. **Solving Problems about Percent Increase or Decrease** — Let's use tape diagrams, equations, and reasoning to solve problems with negatives and percents.

Section C: Inequalities

Section Goals

- Draw and label a graph on a number line that represents all the solutions to an inequality.
- Solve an inequality involving any of the four inequality symbols.

Section Narrative

In this section, students apply what they have learned about equations to inequalities. First, students consider inequalities and determine which values make the inequality true. Next, they compare equations and inequalities that represent the same situation. They see that solving the related equation gives the boundary point between the values that make an inequality true and the values that make it false. Students learn to test values to the left or right of the boundary point to determine which side of the number line contains the solutions to the inequality. Lastly, students write and solve inequalities to represent situations with constraints.

Lessons

1. **Reintroducing Inequalities** — Let’s work with inequalities.
2. **Finding Solutions to Inequalities in Context** — Let’s solve more complicated inequalities.
3. **Efficiently Solving Inequalities** — Let’s solve more complicated inequalities.
4. **Interpreting Inequalities** — Let’s write inequalities.
5. **Modeling with Inequalities** — Let's look at solutions to inequalities.

Section D: Writing Equivalent Expressions

Section Goals

- Apply properties of operations to write an expression with fewer terms that is equivalent to a given expression.
- Apply the distributive property to factor or expand an expression.

Section Narrative

In this section, students work with linear expressions and justify whether they are equivalent. Having learned to perform arithmetic with rational numbers, students are now prepared to examine more complex expressions. First, students see that rewriting subtraction as adding the opposite makes it possible to apply properties of addition, such as the associative or commutative property, to generate equivalent expressions. Next, students apply the distributive property to factor or expand expressions with rational coefficients. They see that combining like terms is an application of the distributive property. Lastly, students combine all these skills to add and subtract linear expressions.

Lessons

1. **Subtraction in Equivalent Expressions** — Let's find ways to work with subtraction in expressions.
2. **Expanding and Factoring** — Let's use the distributive property to write expressions in different ways.
3. **Combining Like Terms (Part 1)** — Let's see how we can tell that expressions are equivalent.
4. **Combining Like Terms (Part 2)** — Let’s see how to use properties correctly to write equivalent expressions.

Section E: Let’s Put It to Work

Section Narrative

In this final section, students have the opportunity to apply their thinking from throughout the unit. As this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions.

Lessons

1. **Applications of Expressions** — Let’s use expressions to solve problems.

Unit 7: Angles, Triangles, and Prisms

Unit Goal

Students investigate angle relationships, conditions for unique triangles, and the area, surface area, and volume of prisms.

Unit Narrative

In this unit, students investigate whether sets of angle and side length measurements determine unique triangles or multiple triangles, or fail to determine triangles. Students also study and apply angle relationships, learning to understand and use the terms “complementary,” “supplementary,” “vertical angles,” and “unique.” The work gives them practice working with rational numbers and equations for angle relationships. Students analyze and describe cross-sections of prisms, pyramids, and polyhedra. They understand and use the formula for the volume of a right rectangular prism and solve problems involving area, surface area, and volume. Students should have access to their geometry toolkits so that they have an opportunity to select and use appropriate tools strategically. Note: It is not expected that students memorize which conditions result in a unique triangle, an impossible-to-create triangle, or multiple possible triangles. Understanding that, for example, side-side-side (SSS) information results in zero or exactly one triangle will be explored in high school geometry. At this level, students should attempt to draw triangles with the given information and notice that there is only one way to do it (or that it is impossible to do).

Section A: Angle Relationships

Section Goals

- Solve multi-step problems involving complementary, supplementary, and vertical angles.
- Write an equation to represent the relationship between angles in a given diagram.

Section Narrative

Students explore angle relationships in this section. They begin by using pattern blocks to determine how many of each type of angle fit around a point or in another known angle, then use that information to determine the angle measures of the pattern blocks. Next, students move to specific angle relationships including complementary and supplementary, and use that understanding to solve for missing angle measures. Then students explore vertical angles and informally use supplementary angle relationships to understand why vertical angles must have equal measures. Students next represent these angle relationships using equations. They put together their understanding of different types of angle relationships with the equation representations to identify and create equations, and then solve for unknown angle measures.

Lessons

1. **Relationships of Angles** — Let’s examine some special angles.
2. **Adjacent Angles** — Let’s look at some special pairs of angles.
3. **Nonadjacent Angles** — Let’s look at angles that are not right next to one another.
4. **Solving for Unknown Angles** — Let’s figure out some missing angles.
5. **Using Equations to Solve for Unknown Angles** — Let’s figure out missing angles using equations.

Section B: Drawing Polygons with Given Conditions

Section Goals

- Draw triangles with two given angle measures and one side length, one given angle measure and two side lengths, or three side lengths.
- Justify whether 3 measures of angles or sides determine a unique triangle or more than one triangle, or if no triangle is possible.

Section Narrative

During these lessons students think about what conditions are needed to determine a unique figure, in preparation for future work with congruence in grade 8 and high school. These lessons continue the language used in grade 6: Two polygons are identical if they match up exactly when placed one on top of the other. Students first experiment with creating polygons using fixed side lengths to notice which conditions make it possible to construct unique shapes or impossible to construct a shape. Next, they generalize a rule for side lengths of triangles as they compare lengths that do and do not create a triangle. This rule is that the sum of any 2 side lengths of a triangle must be greater than the third side's length. Then, students consider possible triangle constructions with given side length and angle combinations and notice patterns of when these combinations create unique triangles and when they do not. They then build on this work to construct triangles with given angle and side length conditions. Students identify patterns of when it is possible to construct unique patterns, although they do not need to know formal definitions for triangle congruence at this time.

Lessons

1. **Building Polygons (Part 1)** — Let's build shapes.
2. **Building Polygons (Part 2)** — Let's build more triangles.
3. **Triangles with 3 Common Measures** — Let's contrast triangles.
4. **Drawing Triangles (Part 1)** — Let's see how many different triangles we can draw with certain measurements.
5. **Drawing Triangles (Part 2)** — Let's draw some more triangles.

Section C: Solid Geometry

Section Goals

- Calculate the surface area and volume of a prism.
- Decide whether to calculate the surface area or volume of a prism to solve a problem in a real-world situation.

Section Narrative

In this section, students find the surface area and volume of right prisms. They begin with making sense of the structure of a prism using cross-sections parallel to the base of the prism. They build on this idea, and on their work in earlier grades with rectangular prisms, to conclude that the volume of any prism can be determined by multiplying the area of the base by the height of the prism. Students then decompose prisms with various types of bases, to continue making sense of the volume of a prism. Then students use nets to understand the parts of the prism as a two-dimensional surface. They associate the lateral surface area with a large rectangle whose height is the height of the prism and whose length is the perimeter of the base of the prism. Students then apply this understanding to finding the surface area of prisms in real-world situations. Finally, students consider whether situations call for finding the surface area or the volume of a given figure and solve these problems in context.

Lessons

1. **Slicing Solids** — Let's see what shapes you get when you slice a three-dimensional object.
2. **Volume of Right Prisms** — Let’s look at volumes of prisms.
3. **Decomposing Bases for Area** — Let’s look at how some people use volume.
4. **Surface Area of Right Prisms** — Let’s look at the surface area of prisms.
5. **Distinguishing Volume and Surface Area** — Let’s work with surface area and volume in context.
6. **Applying Volume and Surface Area** — Let's explore things that are proportional to volume or surface area.

Section D: Let’s Put It to Work

Section Narrative

In this final section, students have the opportunity to apply their thinking from throughout the unit. Because this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions.

Lessons

1. **Building Prisms (Optional)** — Let’s build a triangular prism from scratch.

Unit 8: Probability and Sampling

Unit Goal

Students work with probability and sampling, designing simulations, representing sample spaces, and using samples to make inferences about populations.

Unit Narrative

In this unit, students work with probability and sampling. They use their understanding of basic chance experiments to quantify how likely events are to happen and develop a working understanding of probability. Then they design and use simulations to further understand probability as the frequency of the event occurring when repeating an experiment many times. Students represent sample spaces using tables, tree diagrams, and lists, and use the number of outcomes in a sample space to calculate an expected probability. Next, students examine different ways to collect data from samples within a population to understand why random selection is useful. Then students generate samples and estimate information about the population from sample data. Finally, students compare two groups by examining the measures of center and measures of variability calculated from sample data representing each group.

Section A: Probabilities of Single-Step Events

Section Goals

- Use the results from a repeated experiment to estimate the probability of an event, and justify the estimate.
- Use the sample space to determine the probability of an event, and express it as a fraction, decimal, or percentage.

Section Narrative

In this section, students learn how to quantify the likelihood of events using probability. First, they use their intuition about chance experiments, such as drawing items from a bag or flipping a coin, to informally describe how likely events are to happen. Then, students begin to assign values to the likelihood, such as 50% or a fraction, and recognize these as the fraction of times an event is likely to happen after many repeated trials of the chance experiment. Throughout the section, students encounter different chance experiments, such as drawing from a bag, spinning spinners, or rolling number cubes, that can be used to simulate other experiments. Then, students design their own simulation to estimate the probability of an experiment that would be difficult to repeat.

Lessons

1. **Mystery Bags** — Let's make predictions based on what we know.
2. **Chance Experiments** — Let’s investigate chance.
3. **What Are Probabilities?** — Let’s find out what's possible.
4. **Estimating Probabilities through Repeated Experiments** — Let’s do some experimenting.
5. **More Estimating Probabilities** — Let’s estimate some probabilities.
6. **Estimating Probabilities Using Simulation** — Let’s simulate real-world situations.

Section B: Probabilities of Multi-step Events

Section Goals

- Describe a multi-step experiment that could be used to simulate a compound event in a real-world situation, and justify that it represents the situation.
- Interpret or create a list, table, or tree diagram that represents the sample space of a compound event, and use the sample space to write the probability for an event.

Section Narrative

In this section, students expand their understanding of probability to more complex chance experiments that involve multiple steps, such as rolling a number cube and flipping a coin or predicting the weather on multiple days. They begin by continuing their work with simulations to estimate probabilities of these types of situations, then they use structures such as tree diagrams, tables, and lists to record the sample spaces. After using those sample spaces to write probabilities, they return to simulating chance experiments by designing their own simulations.

Lessons

1. **Simulating Multi-step Experiments** — Let's simulate more complicated events.
2. **Keeping Track of All Possible Outcomes** — Let's explore sample spaces for experiments with multiple parts.
3. **Multi-step Experiments** — Let's look at probabilities of experiments that have multiple steps.
4. **Designing Simulations** — Let's simulate some real-life scenarios.

Section C: Sampling

Section Goals

- Describe methods to obtain a random sample from a population, and explain why it is representative of the population.
- Explain why samples are necessary and describe a sample and population for a given statistical question.

Section Narrative

In this section, students examine samples taken from a larger population and use randomness to select a sample that is less biased than other methods. They begin by comparing two small populations based on their measures of center and variability. Then students consider much larger populations to motivate the need to sample to obtain data. This leads to considering how some samples may be more representative of the population than others and the idea that random selection is more likely to produce representative samples.

Lessons

1. **Comparing Groups** — Let's compare two groups.
2. **Larger Populations** — Let's compare larger groups.
3. **What Makes a Good Sample?** — Let's see what makes a good sample.
4. **Sampling in a Fair Way** — Let's explore ways to get representative samples.

Section D: Using Samples

Section Goals

- Determine whether two populations are likely to be meaningfully different by reasoning about center and spread.

- Use the proportion of a random sample that is within a certain category to make inferences about the population, and explain the reasoning.

Section Narrative

In this section, students use samples to gain information about the populations they represent. In particular, students estimate measures of center and proportions for populations based on information from a sample. Then, they use samples and a general rule involving measures of variability to determine whether the two groups are meaningfully different.

Lessons

1. **Estimating Population Measures of Center** — Let's use samples to estimate measures of center for the population.
2. **Estimating Population Proportions** — Let's estimate population proportions using samples.
3. **More about Sampling Variability (Optional)** — Let's compare samples from the same population.
4. **Comparing Populations Using Samples** — Let's compare different populations using samples.
5. **Comparing Populations with Friends** — Let's ask important questions to compare groups.

Section E: Let's Put It to Work

Section Narrative

In this final section, students have the opportunity to apply their thinking from throughout the unit. As this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions. The lesson in this section is optional because it offers additional opportunities to practice standards that are not a focus of the grade.

Lessons

1. **Memory Test (Optional)** — Let's put it all together.

Unit 9: Putting It All Together

Unit Goal

Students use concepts and skills from previous units to solve problems in the contexts of running a restaurant, making connections across topics, and designing a course.

Unit Narrative

In this optional unit, students use concepts and skills from previous units to solve problems. The first section focuses on calculating or estimating quantities associated with running a restaurant. The second section explores a variety of different contexts, such as population density, Fermi problems, measurement error, and deforestation. In the last section, students build a trundle wheel and design a five-kilometer race course. All related standards in this unit have been addressed in prior units. These sections provide an optional opportunity for students to go more deeply and make connections between domains.

Section A: Running a Restaurant

Section Goals

- Apply proportional reasoning, algebraic expressions, and percentages to model aspects of running a restaurant.
- Justify numerical decisions involved in running a restaurant.

Section Narrative

This optional sequence of lessons uses the context of running a restaurant to review mathematical concepts from previous units. While the lessons in this section are related to each other, they do not need to be completed in order. It is not necessary to do all of the lessons to get some benefit, although more connections are made the further one gets. Students begin this section by using spreadsheets to determine the cost per serving of a recipe. They then use their knowledge of the cost of food as well as other expenses to decide how much money a restaurant needs to earn to make a profit. Finally, students design a scale model of a floor plan of a restaurant, making sure there is enough space around tables but not so much space that food cannot be delivered quickly. All standards in this section have been addressed in prior units. This section provides an optional opportunity for students to go more deeply and make connections between domains.

Lessons

1. **Cost of a Meal (Optional)** — Let's explore how much a recipe's ingredients cost.
2. **Costs of Running a Restaurant (Optional)** — Let's explore how much running a restaurant costs.
3. **Restaurant Floor Plan (Optional)** — Let's design the floor plan for a restaurant.

Section B: Making Connections

Section Goals

- Apply proportional reasoning, algebraic expressions, and percentages to model real-world situations.
- Choose an appropriate level of accuracy when reporting estimates of quantities.

Section Narrative

These optional lessons do not build on each other and can be completed in any order. This section begins with two lessons that focus on proportional reasoning. The next two lessons involve writing algebraic expressions representing real-world situations. The final lesson explores decomposing regions and using scale drawings. Each lesson builds on skills developed in earlier units. All standards in this section have been addressed in prior units. This section provides an optional opportunity for students to go more deeply and make connections between domains.

Lessons

1. **How Crowded Is This Neighborhood? (Optional)** — Let's see how proportional relationships apply to where people live.
2. **Fermi Problems (Optional)** — Let's estimate some quantities.
3. **More Expressions and Equations (Optional)** — Let's solve harder problems by writing equivalent expressions.
4. **Measurement Error (Optional)** — Let's check how accurate our calculations are.
5. **Deforestation at Scale (Optional)** — Let's compare the forested area over time.

Section C: Designing a Course

Section Goals

- Calculate the distance of a path using the circumference and number of rotations of a trundle wheel.
- Compare and contrast methods for measuring long distances.

Section Narrative

This optional sequence of lessons can be done anytime after Unit 4 is completed. In this section, students explore different ways to measure long distances and then design a five-kilometer course for their school. They begin by measuring a smaller track using a variety of tools and strategies. Then they are introduced to a trundle wheel, a tool for measuring distance on uneven or curving surfaces. Students build their own trundle wheels and compare this method of measuring with their earlier choices. Finally, students use a map of the school grounds to plan a five-kilometer course and use their trundle wheels to verify its length. The lessons in this section build on each other and should be completed in order. It is not necessary to do all of the lessons to get some benefit, although more connections are made the further one gets. All standards in this section have been addressed in prior units. This section provides an optional opportunity for students to go more deeply and make connections between domains.

Lessons

1. **Measuring Long Distances over Uneven Terrain (Optional)** — Let's measure long distances over uneven terrain.
2. **Building a Trundle Wheel (Optional)** — Let's build a trundle wheel.
3. **Using a Trundle Wheel to Measure Distances (Optional)** — Let's use our trundle wheels.
4. **Designing a 5K Course (Optional)** — Let's map out the 5K course.

About This Syllabus

This syllabus was compiled from the publicly available Illustrative Mathematics 6-8 v.360 curriculum at accessim.org. Unit and section narratives, goals, and lesson titles are sourced from the teacher view. Optional lessons are labeled “(Optional).”
Assessments (checkpoints and end-of-unit assessments) are referenced in the source curriculum but require a Kendall Hunt educator account to view; they are not included here.
https://accessim.org/6-8/grade-7?a=teacher