Accelerated 6 Mathematics

Full Syllabus

Illustrative Mathematics® 6-8 Accelerated Math (v.360)

9 Units · Unit Goals · Section Goals · Lesson-by-Lesson Outline

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Course Overview

Accelerated 6 of the Illustrative Mathematics 6-8 Accelerated v.360 curriculum is organized into 9 units. It compresses the content of grade 6 and the first part of grade 7 into a single course. Students reason about areas of polygons and surface area; explore ratios, rates, and percentages; develop fraction and decimal operations; write and solve equations and expressions; investigate proportional relationships, including circumference and area of circles; apply percent increase and decrease; extend the number system to rational numbers; study data sets, distributions, sampling, and probability; and culminate with an optional Putting It All Together unit.

The unit sequence is:

- Unit 1: Areas
- Unit 2: Ratios, Rates, and Percentages
- Unit 3: Fractions and Decimals
- Unit 4: Equations and Expressions
- Unit 5: Proportional Relationships
- Unit 6: Percent Increase and Decrease
- Unit 7: Rational Numbers
- Unit 8: Data Sets, Distributions, and Sampling
- Unit 9: Putting It All Together

Each unit is divided into lettered sections (A, B, C, D, etc.), and each section is a sequence of daily lessons. Optional lessons are noted with “(Optional).”

Unit 1: Areas

Unit Goal

Students reason about areas of polygons and surface areas of polyhedra, building on geometric understandings developed in earlier grades.

Unit Narrative

In this unit, students reason about areas of polygons and surface areas of polyhedra, building on geometric understandings developed in earlier grades. In grade 3, students found the area of rectangles with whole-number side lengths. They also found the area of rectilinear figures by decomposing them into non-overlapping rectangles and adding those areas. Students used a formula for the area of rectangles in grade 4 and found the area of rectangles with fractional side lengths in grade 5. In this unit, students extend their reasoning about area to include shapes that are not composed of rectangles. They use strategies such as decomposing and rearranging to find areas of parallelograms and generalize their process as a formula. Their work with parallelograms then becomes the basis for finding the area of triangles. Students see that other polygons can be decomposed into triangles and use this knowledge to find areas of polygons. Next, students calculate the surface areas of polyhedra with triangular and rectangular faces. They study, assemble, and draw nets of prisms and pyramids and use nets to determine surface areas. In many lessons, students engage in geometric work without a context. This design choice is made in recognition of the significant intellectual work of reasoning about area. Later in the unit, students have opportunities to apply their learning in context.

Section A: Reasoning to Find Area

Section Goals

- Find the area of a two-dimensional region with straight boundaries by decomposing, rearranging, subtracting, or enclosing shapes, and explain the solution method.

Section Narrative

In this section, students explore strategies for reasoning about the area of two-dimensional figures. The explorations highlight two principles about area: figures that match exactly have equal areas, and area is additive. First, students compare the amount of the plane covered by different shapes by thinking about the relationships between the shapes. They also recall their understanding of what area is, refine it, and develop a shared definition. Next, students use their definition of “area” and strategies, such as composing, decomposing, and rearranging, to reason about areas of tangram shapes. They create figures with certain areas and find the area of given shapes. Finally, students apply these reasoning strategies to find the areas of figures drawn on and off a grid.

Lessons

1. **Tiling the Plane** — Let’s look at tiling patterns and think about area.
2. **Finding Area by Decomposing and Rearranging** — Let’s create shapes and find their areas.
3. **Reasoning to Find Area** — Let’s decompose and rearrange shapes to find their areas.

Section B: Parallelograms

Section Goals

- Apply the formula for area of a parallelogram to find the area, the length of a base, or the height.

- Understand bases and heights in a parallelogram and recognize base-height pairs to use to calculate its area.

Section Narrative

In this section, students reason about areas of parallelograms. They learn about bases and heights and analyze the measurements that can be used to find the area of any parallelogram. First, students use strategies they learned earlier in the unit to find the areas of given parallelograms. They see that one way to find the area of a parallelogram is by decomposing it and rearranging the pieces into a rectangle. Another way is to enclose it in a rectangle and subtract the areas of the extra pieces. Along the way, students notice regularity in both the process of finding area (that it can be done using one or more related rectangles) and in the measurements that are useful for finding area (that they are side lengths of the related rectangles). Students make sense of these lengths as "bases" and "heights" of a parallelogram and learn to identify them. Then, students generalize the process of using bases and heights to find areas, express it as a formula, and use the formula to find the area of any parallelogram and to solve problems.

Lessons

1. **Parallelograms** — Let's investigate the characteristics and area of parallelograms.
2. **Area of Parallelograms** — Let's investigate the area of parallelograms some more.
3. **From Parallelograms to Triangles** — Let's compare parallelograms and triangles.

Section C: Triangles and Other Polygons

Section Goals

- Find the area of a polygon by decomposing it into parallelograms and triangles.
- Understand bases and heights in a triangle and recognize base-height pairs to use to find the area of a triangle.
- Understand why the process of finding the area of a triangle can be abstracted as one-half base times height (or equivalent) and apply the formula to find the area of a triangle.

Section Narrative

In this section, students explore ways to find areas of triangles, generalize their observations as a formula, and use the formula to find the area of any triangle. They also apply their insights regarding triangles and parallelograms to find areas of other polygons. Students begin by investigating the relationship between triangles and parallelograms. They see that a parallelogram can always be decomposed into two identical triangles. Likewise, two copies of any triangle can be composed into a parallelogram. Next, students learn that triangles also have bases and heights, which correspond to those in a related parallelogram. They observe that the area of a triangle is half that of a related parallelogram that shares the same base and height. Students generalize their observations with the expression one-half times base times height and use it to solve problems.

Lessons

1. **Area of Triangles** — Let's use what we know about parallelograms to find the area of triangles.
2. **Formula for the Area of a Triangle** — Let's write and use a formula to find the area of a triangle.
3. **Polygons** — Let's investigate polygons and their areas.

Section D: Surface Area

Section Goals

- Identify or create a net that represents a prism or pyramid.
- Use a net to calculate the surface area of a prism or pyramid and explain the solution method.

Section Narrative

This section introduces students to polyhedra and surface area. Students apply their knowledge about areas of polygons to create nets and find surface areas of three-dimensional figures. First, students learn that surface area is the number of unit squares that covers all the faces of a three-dimensional figure, without gaps or overlaps. They reason about the surface areas of rectangular prisms and figures built from unit cubes. Then students explore the characteristics of a polyhedron and develop a working definition for one. They learn that a net is a two-dimensional representation of a polyhedron. Students assemble prisms and pyramids from nets and also visualize the polyhedron that can be assembled from a given net. Later, students create nets for given prisms or pyramids and use the nets to calculate the surface areas. The section includes an optional lesson to help reinforce students' understanding of surface area and volume as distinct attributes of three-dimensional figures.

Lessons

1. **What Is Surface Area?** — Let's cover the surfaces of some three-dimensional objects.
2. **Polyhedra and Nets** — Let's use nets to find the surface area of polyhedra.
3. **More Nets, More Surface Area** — Let's draw nets and find the surface area of polyhedra.

Section E: Let's Put It to Work

Section Narrative

In this final section, students have the opportunity to apply their thinking from throughout the unit. As this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions. The lesson in this section is optional because it offers additional opportunities to practice standards that are not a focus of the grade.

Lessons

1. **All about Tents (Optional)** — Let's find out how much material is needed to build some tents.

Unit 2: Ratios, Rates, and Percentages

Unit Goal

Students develop an understanding of ratios, equivalent ratios, rates, unit rates, and percentages, and use representations such as discrete diagrams, double number lines, tables, and tape diagrams to solve problems.

Unit Narrative

This unit introduces students to ratios, rates, and percentages. It builds on previous experiences students had with relating two quantities, such as converting measurements and finding equivalent fractions starting in grade 3, multiplicative comparison in grade 4, and interpreting multiplication as scaling in grade 5. The work prepares students to reason about proportional relationships, constants of proportionality, and percent increase and decrease later in this course. The first half of the unit focuses on ratios and equivalent ratios. Students learn that a ratio is an association between two quantities. They use sentences, drawings, or discrete diagrams to represent ratios that describe collections of objects and recipes. Next, students encounter equivalent ratios in terms of multiple batches of a recipe. They learn to use double number line diagrams and tables to represent and reason about equivalent ratios. Students explore equivalent ratios in contexts, such as constant speed and uniform pricing. The second half of the unit focuses on rates and percentages. Students begin by recalling what they know about standard units of measurement—the attributes that they measure and their relative sizes. They use ratios and rates to reason about measurements and to convert between units of measurement. Next, students learn about unit rates. They see that there are two unit rates associated with any ratio and interpret them in context. Students then use their understanding of ratios and rates to make sense of percentages. Just as a unit rate can be interpreted in context as a rate per 1, a percentage can be interpreted in context as a rate per 100.

Section A: What Are Ratios?

Section Goals

- Create and interpret discrete diagrams that represent situations involving ratios.
- Explain equivalent ratios in terms of the quantities in a recipe being multiplied by the same number to create a different size batch of something with the same taste or same shade.
- Generate equivalent ratios and justify that they are equivalent.

Section Narrative

This section introduces students to ways to represent ratios and the concept of equivalent ratios. Students begin by using ratios and ratio language to describe collections of physical objects. Next, students draw diagrams to represent situations involving one or more ratios. They learn that simple diagrams can be useful and efficient for reasoning about ratios. Students then make sense of equivalent ratios through concrete experiences involving recipes. They learn that scaling a recipe up or down—to create multiple batches or a fraction of a batch—produces a result that is the same as the original recipe in some important way. In each case, students see that scaling a recipe involves multiplying each quantity by the same factor, producing a ratio of ingredients that is equivalent to that in the original recipe.

Lessons

1. **Representing Ratios with Diagrams** — Let's use diagrams to represent ratios.
2. **Mixtures** — Let's explore what mixing has to do with ratios.

3. **Defining Equivalent Ratios** — Let's investigate equivalent ratios some more.

Section B: Representing Equivalent Ratios

Section Goals

- Choose multipliers strategically when solving multi-step problems involving equivalent ratios.
- Use a double number line diagram to represent and find equivalent ratios.
- Use a table of equivalent ratios to solve problems.

Section Narrative

This section introduces new ways to represent and describe equivalent ratios. Students see that double number line diagrams are useful for reasoning about equivalent ratios. Students reason about equivalent ratios where one quantity has a value of 1 and express them using language such as "for 1" and "per." They relate the phrases "at this rate" and "at the same rate" to equivalent ratios in contexts, such as constant speed, uniform pricing, and recipes. Later in the section, students use tables to reason about situations involving equivalent ratios. They see that the values in a table don't need to be listed in order, so they can choose the multipliers strategically. This makes a table a more flexible reasoning tool than a double number line. While certain ways of working with tables are presented, students can use other representations for support. They should be encouraged to explain their choice and compare the efficiency of different methods.

Lessons

1. **Introducing Double Number Line Diagrams** — Let's use number lines to represent equivalent ratios.
2. **Creating Double Number Line Diagrams** — Let's draw double number line diagrams like a pro.
3. **How Much for One?** — Let's use ratios to describe how much things cost.
4. **Constant Speed** — Let's use ratios to work with how fast things move.
5. **Comparing Situations by Examining Ratios** — Let's use ratios to compare situations.
6. **Representing Ratios with Tables** — Let's use tables to represent equivalent ratios.
7. **Navigating a Table of Equivalent Ratios** — Let's use a table of equivalent ratios like a pro.
8. **Solving Equivalent Ratio Problems** — Let's practice getting information from our partner.

Section C: Part-Part-Whole Ratios

Section Goals

- Choose representations and solution methods to reason about ratios and sums of quantities.
- Use diagrams or other strategies to solve problems involving ratios and the total amount.

Section Narrative

In this section, students reason about situations in which the quantities in the ratio have the same units and questions can be asked about the individual quantities (the parts) and their sum (the total). Students learn to use tape diagrams as a way to represent such situations. They also interpret ratios expressed in "parts" rather than standard units, such as cups, ounces, meters, and so on. As they solve application problems, students have greater flexibility to choose reasoning strategies and representations (such as double number lines, tables, or tape diagrams) that make sense to them and to the problem at hand.

Lessons

1. **Part-Part-Whole Ratios** — Let's look at situations where you can add the quantities in a ratio together.
2. **Solving More Ratio Problems** — Let's compare all our strategies for solving ratio problems.

Section D: Units of Measurement and Unit Conversion

Section Goals

- Recognize that when we measure things in two different units, the pairs of measurements are equivalent ratios. Apply this understanding to convert a measurement from one unit to another unit.
- Use a "rate per 1" to solve problems involving unit conversion.

Section Narrative

This section extends students' knowledge of units of measurement. It prompts them to reason about ratios and rates to perform unit conversion, including across different measurement systems. Students begin by grounding their perception of standard measurement units in benchmark objects. For instance, they relate 1 foot to the length of a ruler and 1 milliliter to the volume of liquid in an eyedropper. This experience builds students' intuition for the relative sizes of various units, preparing them to better see relationships between units as ratios. Next, students recall that it takes more of a smaller unit than a larger unit to measure the same quantity. Although this idea is a foundation for converting units by reasoning about ratios, the lesson is optional because it revisits ideas from earlier grades. In the last lesson of the section, students learn that two measurements of the same thing given in different units form a ratio. They see that conversion across units can be done by finding equivalent ratios and using familiar representations, and how "rates per 1" can be helpful in solving problems.

Lessons

1. **Anchoring Units of Measurement** — Let's see how big different things are.
2. **Measuring with Different-Size Units (Optional)** — Let's measure things.
3. **Converting Units** — Let's convert measurements to different units.

Section E: Rates

Section Goals

- Apply reasoning about ratios and unit rates to solve problems and explain the solution methods.
- Calculate the two unit rates associated with a ratio and interpret them in the context of a situation.

Section Narrative

In this section, students further explore "rates per 1" and solve various problems involving rates. Students observe that using rates per 1 is a helpful way to make comparisons. They learn that a rate per 1 is a "unit rate," and that each ratio has two associated unit rates. In interpreting unit rates in context, students see that one unit rate might be more helpful than the other, depending on the question. Students generalize that when two ratios have the same unit rate, the ratios are equivalent. Likewise, when ratios are equivalent, they have the same unit rates. In a table of equivalent ratios, each unit rate is a factor that relates the values in the two columns. As students progress through the section, they encounter problems with less scaffolding. In the last lesson, students reason about the movement of

two objects relative to each other. This lesson is optional because it exceeds the expectations of the standards.

Lessons

1. **Comparing Speeds and Prices** — Let's compare some speeds and some prices.
2. **Interpreting Rates** — Let's explore unit rates.
3. **Equivalent Ratios Have the Same Unit Rates** — Let's revisit equivalent ratios.
4. **Solving Rate Problems** — Let's investigate the Burj Khalifa building.

Section F: Percentages

Section Goals

- Choose a strategy to solve problems involving percentages and explain the solution method.
- Create tape diagrams, double number line diagrams, or tables to represent situations involving percentages.

Section Narrative

In this section, students make sense of percentages as rates per 100 and solve problems involving percentages. Students begin by reasoning about percentages of 100 and of 1. Then they work with percentages of other quantities, paying attention to what 100% represents in each situation. To reinforce percentages as rates, double number line diagrams are the primary representation used initially. Later, students also make sense of percentages using tables and tape diagrams. Tape diagrams help connect benchmark percentages (10%, 25%, 50%, and 75%) to benchmark fractions. Students go on to solve various problems involving percentages. When they encounter numbers that are difficult to manipulate mentally or by using diagrams, students generalize the process for calculating any percentage of any number. In the last lesson, students find a general way to determine what percentage a number is of another number. Generalizing this calculation is beyond the expectations of the course, so this lesson is optional.

Lessons

1. **What Are Percentages?** — Let's use double number line diagrams to understand percentages.
2. **Representing Percentages in Different Ways** — Let's use tape diagrams to understand percentages.
3. **Benchmark Percentages** — Let's contrast percentages and fractions.
4. **Solving Percentage Problems** — Let's solve more percentage problems.
5. **Finding This Percent of That** — Let's solve percentage problems like a pro.
6. **Finding the Percentage (Optional)** — Let's find percentages in general.

Section G: Let's Put It to Work

Section Narrative

In the final section, students have the opportunity to apply their thinking from throughout the unit. As this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions. All lessons in this section are optional.

Lessons

1. **A Fermi Problem** — Let's solve a Fermi problem.

2. **Painting a Room** — Let’s see what it takes to paint a room.

Unit 3: Fractions and Decimals

Unit Goal

Students develop understanding of division of fractions by fractions and operations on whole numbers and decimals, including addition, subtraction, multiplication, and division.

Unit Narrative

This unit develops students' understanding of division of fractions by fractions and operations on whole numbers and decimals. This work draws on students' prior knowledge of multiplication, division, and the relationship between the two. It also builds on place value, properties of operations, and the relationship between addition and subtraction. Students begin by exploring meanings of division and the relationship between the quantities in division situations. They recall that we can think of dividing as finding an unknown factor in a multiplication equation. In situations involving equal-size groups, division can be used to answer two questions: "How many groups?" and "How much in each group?" Students investigate ways to answer those two questions. Students then apply their insights to generalize the process of finding quotients. They notice that dividing a number by a fraction is the same as multiplying that number by its reciprocal. Students go on to use this algorithm to solve problems about geometric figures that have fractional length, area, or volume measurements and to compute unit rates of fractions. Next, students revisit addition and subtraction of decimals, using both concrete representations and numerical calculations. They also investigate various ways to find the product of two decimals: using decimal fractions, using diagrams and partial products, and reasoning about the relationship between a decimal and a related whole number. The next section focuses on division. Students have an opportunity to use base-ten blocks or diagrams to represent division of multi-digit numbers before exploring other numerical methods, such as using partial quotients and long division. Students progress through calculations of increasing complexity. A deeper understanding of multiplication, division, and ways to represent them will support students in reasoning about writing and solving variable equations later in the course.

Section A: Making Sense of Division

Section Goals

- Create a diagram and a multiplication or division equation to represent the relationship in situations involving equal-size groups, and coordinate these representations.
- Interpret a division expression in two ways: as an answer to a "How many groups?" question or to a "How many in each group?" question.

Section Narrative

In this section, students investigate the quantities in a division situation, recall the relationship between multiplication and division, and revisit ways to represent this relationship. First, students look at how the numbers in a division situation are related. Students observe—in concrete and abstract situations—how the size of the dividend and the divisor affect the size of the quotient. Next, students reason about division in terms of equal-size groups. They are reminded that multiplication is a way to find the total amount when given the number of groups and the size of each group, and division is a way to find an unknown factor when given a total amount. This means division can be interpreted in two ways. The last lesson in the section is optional. It offers additional practice in interpreting and representing situations that involve equal-size groups.

Lessons

1. **Size of Divisor and Size of Quotient** — Let's explore quotients of different sizes.
2. **Meanings of Division** — Let's explore ways to think about division.

Section B: Dividing Fractions

Section Goals

- Generalize a process for dividing a number by a fraction.

Section Narrative

In this section, students extend their understanding of division to include division by fractions. Next, students explore cases in which the size of one group is unknown. They learn that questions such as "If there are 3 pounds in some bags, how much is in 1 bag?" and "some fraction of what number is 4?" can also be answered by dividing and can be represented with diagrams and equations. An optional lesson gives practice finding the amount in one group and solving various division problems in context. As students use tape diagrams to reason repeatedly about division by a fraction, they look for structure and regularity in the process. Students generalize that to divide a number by a non-unit fraction is to figure how many unit fractions are in the dividend and then see how many groups of unit fractions can be formed. Finally, students practice applying this algorithm and using it strategically to solve problems involving constant rates between fractional quantities.

Lessons

1. **How Many Groups?** — Let's draw tape diagrams to think about division with fractions.
2. **What Fraction of a Group?** — Let's think about dividing things into groups when we can't even make one whole group.
3. **How Much in Each Group? (Part 1)** — Let's look at division problems that help us find the size of one group.
4. **How Much in Each Group? (Part 2) (Optional)** — Let's practice dividing fractions in different situations.
5. **Finding an Algorithm for Dividing Fractions** — Let's look for patterns when we divide by a fraction.
6. **Ratios and Rates with Fractions** — Let's calculate some rates with fractions.

Section C: Fractions in Lengths, Areas, and Volumes

Section Goals

- Determine the volume of a rectangular prism by packing it with unit cubes with appropriate unit-fraction edge lengths and by multiplying the edge lengths.
- Solve problems about length comparison, the side lengths and area of a rectangle, and the edge lengths and volume of a rectangular prism using fraction division.

Section Narrative

In this section, students apply their insights about multiplication and division, as well as prior knowledge about area and volume, to solve geometric problems. Students first solve multiplicative comparison problems in contexts that involve fractional lengths in one dimension. They use their understanding of division to answer questions such as "How many times as tall or as far is this as that?" Next, students reason about measurements of two-dimensional figures. They find the area of rectangles and triangles given base-height pairs and find an unknown base or height given the area and a known height or base.

Students then go on to reason about three-dimensional figures. They multiply and divide to find the volume of rectangular prisms with fractional edge lengths and to find an edge length when the volume of a prism and two other edge lengths are known. The work in this section builds on geometric concepts from grade 5, during which students found the area of rectangles with fractional side lengths and the volume of rectangular prisms with whole-number edge lengths.

Lessons

1. **Fractional Lengths** — Let's solve problems about fractional lengths.
2. **Rectangles and Triangles with Fractional Lengths** — Let's explore rectangles and triangles that have fractional measurements.
3. **Volume of Prisms** — Let's look at the volume of prisms that have fractional measurements.
4. **Solving Problems Involving Fractions (Optional)** — Let's add, subtract, multiply, and divide fractions.

Section D: Adding, Subtracting, and Multiplying Decimals

Section Goals

- Calculate the product of two decimals and explain the solution method.
- Explain the sum or difference of two decimals in terms of combining like base-ten units and composing (or decomposing) a larger unit from (or into 10) units of a lower value.
- Use the standard algorithm to add or subtract decimals with multiple non-zero digits.

Section Narrative

This section deepens students' understanding of the base-ten structure and extends their ability to add, subtract, and multiply decimals beyond hundredths. Students begin by revisiting decimal operations in the context of money. They make sense of place-value relationships and use base-ten diagrams to represent decimals, their sums, and their differences. Students recall that each place of the base-ten system has a value that is 10 times that of the place to its right. This means that 10 of a base-ten unit can be composed into 1 larger unit, and 1 base-ten unit can be decomposed into 10 of a smaller unit. Students reason about the composition and decomposition using both base-ten diagrams and vertical calculations (or standard algorithm). They apply their understanding to add and subtract decimals, regrouping the values as needed. Next, students compare several methods for multiplying decimals. They express decimal factors as fractions, multiply the fractions, and rewrite the products as decimals. They also make use of the structure of base-ten numbers. Then students reason using area diagrams. They represent the factors as the side lengths of a rectangle, decompose them by place value, find the partial products, and add them to find the product of the original factors. In the last lesson of the section, students generalize the various ways of reasoning into an algorithm.

Lessons

1. **Using Decimals in a Shopping Context** — Let's use what we know about decimals to make shopping decisions.
2. **Using Diagrams to Represent Addition and Subtraction** — Let's represent addition and subtraction of decimals.
3. **Adding and Subtracting Decimals with Many Non-Zero Digits** — Let's practice adding and subtracting decimals.
4. **Methods for Multiplying Decimals** — Let's look at some ways we can represent multiplication of decimals.

5. **Calculating Products of Decimals** — Let's multiply decimals.

Section E: Dividing Decimals

Section Goals

- Calculate the quotient of a whole-number or a decimal dividend and a decimal divisor, and explain the solution method.
- Use long division to divide a multi-digit whole number or a decimal by a whole number.

Section Narrative

In this section, students reason about quotients of multi-digit numbers and work toward the standard algorithm for division. The section begins with an optional lesson that revisits concepts from grade 5 and reinforces the role of place value in division. Students use base-ten diagrams to represent division and use strategies based on place value and properties of operations to find quotients. Next, students calculate quotients by finding partial quotients, likewise building on a strategy developed in earlier courses. Students learn to organize their calculations of partial quotients vertically and to record the steps in a systematic way, which paves the way to long division. Then students make sense of the standard division algorithm by analyzing worked examples, interpreting and explaining the numbers in each step. They learn that to use long division is essentially to find the partial quotients, but doing so one digit at a time and relying on the place of the digit to convey its value. Students practice using the algorithm—first to divide a whole number by a whole number, and then to divide a decimal by a whole number. Finally, students use equivalent division expressions and the algorithm to divide a decimal by a decimal.

Lessons

1. **Using Long Division** — Let's divide whole numbers.
2. **Dividing Numbers that Result in a Decimal** — Let's find quotients that are not whole numbers.
3. **Dividing a Decimal by a Decimal** — Let's divide a decimal by a decimal.
4. **Solving Problems Involving Decimals** — Let's solve some problems using decimals.

Section F: Let's Put It to Work

Section Narrative

In this final section, students have the opportunity to apply their thinking from throughout the unit. As this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions. All lessons in this section are optional.

Lessons

1. **Fitting Boxes into Boxes** — Let's use what we learned about fractions to find shipping costs.
2. **Making and Measuring Boxes (Optional)** — Let's use what we know about decimals to make and measure boxes.

Unit 4: Equations and Expressions

Unit Goal

Students apply their understanding of arithmetic to reason about algebraic expressions and equations, including writing and solving equations, identifying equivalent expressions, and using exponents.

Unit Narrative

In this unit, students apply their understanding of arithmetic to reason about algebraic expressions and equations. In the first section, students work with equations of the form $x + p = q$ and $px = q$ where p and q are positive rational numbers. They use tape diagrams and hanger diagrams to reason about the meaning of equations, and to develop an understanding that to solve an equation is to find a value that would make the equation true. Students end the section by identifying, interpreting, and writing equations to represent and solve real-world problems. In the second section, students write algebraic expressions and evaluate them for given values. They identify and write equivalent expressions, reasoning using diagrams, the distributive property, and other properties of operations. The third section is all about exponents. First, students learn to use exponents 2 and 3 to express areas of squares and volumes of cubes and their units. Next, they write expressions with a whole-number exponent and a base that may be a whole number, a fraction, or a variable. They analyze such expressions for equivalence, as well as use the conventional order of operations to evaluate them. Students also identify solutions to simple exponential equations. In the last two sections, students analyze real-world relationships between two quantities where one quantity depends on the other. They use tables, graphs, and equations to represent and reason about such relationships. The work here prepares students to work with proportional relationships in a later unit, as well as to solve equations that are more complex in grade 7.

Section A: Equations in One Variable

Section Goals

- Solve equations of the form $x + p = q$ or $px = q$ and explain the solution method.
- Understand that solving an equation with a variable means finding a value for the variable that makes the equation true, and use substitution to determine whether a number is a solution to the equation.

Section Narrative

In this section, the key understandings are that an equation with a variable represents a relationship that can be true or false, and a solution to the equation is a value for the variable that makes the equation true. Relationships are limited to those that can be expressed with $x + p = q$ or $px = q$. First, students revisit tape diagrams and equations as tools for representing relationships and learn about variables and solutions. To find solutions to equations, students reason about the numbers and relationships—with or without tape diagrams, and use substitution to determine if a given value makes the equation true. They also build on their understanding that equations are not always true. Next, hanger diagrams are used to help students understand the concept of keeping equations balanced. Students use repeated calculations and make use of structure to generalize an algebraic method for solving equations. They then choose the best strategies to solve equations as well as real-world problems that can be represented by such equations. In the last lesson of the section, students discover that equations of the form $px = q$ can be used to represent and solve percentage problems.

Lessons

1. **Tape Diagrams and Equations** — Let's recall how tape diagrams and equations can show relationships between amounts.
2. **Truth and Equations** — Let's see what it means to find a solution for an equation with a variable.
3. **Staying in Balance** — Let's use balanced hangers to help us solve equations.
4. **Practice Solving Equations** — Let's solve equations in different ways.
5. **Represent Situations with Equations** — Let's use equations to represent and solve real-world problems.

Section B: Equal and Equivalent

Section Goals

- Justify whether two expressions are “equivalent,” or equal to each other for every value of their variable.
- Use the distributive property to write equivalent algebraic expressions.

Section Narrative

In this section, students focus on writing expressions and equivalent expressions. This is also where the distributive property is formally introduced. First, students write expressions that represent situations, using tape diagrams for reasoning as needed. Then equivalent expressions are introduced. Students use tape diagrams to identify when expressions are equal. Students learn that equivalent expressions with variables are always equal when the same value is used for the variable in each expression. They use properties of operations to identify and write equivalent expressions. In prior grades, the names of the properties were not emphasized. In this grade, the names of the properties are used, and students can use them to justify equivalent expressions. However, students may continue to use more informal explanations that demonstrate an understanding of the properties. In the latter half of the section, students explore the distributive property. They use rectangular diagrams to first represent equivalent numerical expressions, and then to reason about equivalent expressions with variables. The final optional lesson offers additional reinforcement in using the distributive property to identify and write equivalent expressions.

Lessons

1. **Write Expressions with Variables** — Let's use expressions with variables to describe situations.
2. **Equal and Equivalent** — Let's use diagrams to figure out which expressions are equivalent and which are just sometimes equal.
3. **The Distributive Property, Part 1** — Let's use the distributive property to describe expressions.
4. **The Distributive Property, Part 2** — Let's practice writing equivalent expressions by using the distributive property.
5. **The Distributive Property, Part 3 (Optional)** — Let's practice writing equivalent expressions by using the distributive property.

Section C: Expressions with Exponents

Section Goals

- Evaluate expressions with whole-number exponents at specific values of their variables.

- Interpret and write expressions with exponents 2 and 3 to represent the area of a square or the volume of a cube.
- Justify whether numerical expressions involving whole-number exponents are equal by evaluating the expressions and performing operations in the conventional order.

Section Narrative

In this section, students extend their understanding of exponential expressions. They learn to evaluate expressions that include both variables and exponents. Students first explore perfect squares as areas of squares and perfect cubes as volumes of cubes with whole-number edge lengths. Next, they learn that the exponents 2 and 3 can be used to express the multiplication of edge lengths of these figures. The exponents 2 and 3 can also be used to express square units and cubic units. Students practice using and interpreting this new notation in the context of squares and cubes. Then students work with whole-number exponents beyond 2 and 3. First, students use exponent notation to represent repeated multiplication in patterns and situations. Next, they use prior knowledge about operations to practice evaluating exponential expressions with various bases, including whole numbers, fractions, and decimals. Next, students evaluate expressions that include exponents and another operation. A real-world example involving surface area of a cube helps students see a rationale for following the order of operations. Then students practice following the conventional order for evaluating expressions with and without grouping symbols. Finally, students encounter exponential expressions with variables and evaluate such expressions at specific values of the variables.

Lessons

1. **Squares and Cubes** — Let’s investigate perfect squares and perfect cubes.
2. **Meaning of Exponents** — Let’s see how exponents show repeated multiplication.
3. **Expressions with Exponents** — Let's use the meaning of exponents to decide if equations are true.
4. **Evaluating Expressions with Exponents** — Let’s find the values of expressions with exponents.
5. **Equivalent Exponential Expressions** — Let's investigate expressions with variables and exponents.

Section D: Relationships Between Quantities

Section Goals

- Create a table, graph, and equation to represent the relationship between two quantities.
- Identify the independent and dependent variable in a situation.

Section Narrative

In this section, students apply their understanding of algebraic expressions and equations to represent two quantities that change in relationship to one another. Students explore these relationships through multiple representations, such as tables, graphs, and equations. They also delve into the concepts of independent and dependent variables. First, students create and analyze tables and graphs that represent how one quantity affects another. The choice of independent and dependent variables is driven by the context or the modeler. Next, the context of traveling at a constant rate is used to create representations of the relationship between time and distance. Students explore how switching independent and dependent variables affects the different representations. Finally, an optional lesson invites students to explore additional contexts in which two quantities vary together, such as in the relationship of side lengths of a rectangle with a constant area and in a doubling relationship, and allows for student choice in representation methods.

Lessons

1. **Two Related Quantities, Part 1** — Let's use tables, equations, and graphs to describe relationships.
2. **Two Related Quantities, Part 2** — Let's use equations and graphs to describe stories with constant speed.

Section E: Let's Put It to Work

Section Narrative

In this final section, students have the opportunity to apply their thinking from throughout the unit. As this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions. All lessons in this section are optional.

Lessons

1. **Tables, Equations, and Graphs, Oh My! (Optional)** — Let's explore some equations from real-world situations.

Unit 5: Proportional Relationships

Unit Goal

Students develop the idea of a proportional relationship and apply proportional reasoning to circumference and area of circles.

Unit Narrative

In this unit, students develop the idea of a proportional relationship. They work with proportional relationships that are represented in tables, as equations, and on graphs. This builds on previous work with equivalent ratios and helps prepare students for the study of linear functions in later courses. In a table of equivalent ratios, a multiplicative relationship between a pair of rows is given by a scale factor, while the multiplicative relationship between the columns is given by a unit rate. Students learn that the relationship between pairs of values in the two columns is called a "proportional relationship," and the unit rate that describes this relationship is called a "constant of proportionality." Students use equations of the form y = kx to represent proportional relationships and solve problems. They determine whether given tables and equations could represent a proportional relationship. Then students investigate graphs of proportional relationships. They recognize that the graph of a proportional relationship is a straight line through the origin. They interpret points on the graph, including the point (1, k). Next, students apply their knowledge of proportional relationships to the context of measuring circles. This builds on students' work from previous grades with perimeter and area of polygons. The terms "center," "radius," "diameter," and "circumference" are introduced. Then students investigate the relationship between circumference and diameter and see that it is a proportional relationship. They apply this relationship to solve problems. Next, students explore the area of circular regions. They see an informal derivation that shows where the formula A = pi r squared comes from and then use this formula to solve problems.

Section A: Representing Proportional Relationships with Equations

Section Goals

- Use an equation to solve problems involving a proportional relationship.
- Write an equation of the form y = kx to represent a proportional relationship, given a table or a description of the situation.

Section Narrative

In this section, students use equations to represent proportional relationships and solve problems. They learn that any proportional relationship can be represented by an equation of the form y = kx, where k is the constant of proportionality. Students begin by revisiting some of the same contexts that they previously examined with tables. They think about how the repeated calculations can be expressed with an equation. Next, students see that there are two different equations that represent each situation, depending on which quantity is regarded as being proportional to the other. The two constants of proportionality in those two equations are reciprocals of each other. Then students write equations and use them to solve problems involving proportional relationships in new contexts.

Lessons

1. **Proportional Relationships and Equations** — Let's write equations describing proportional relationships.
2. **Two Equations for Each Relationship** — Let's investigate equations that represent proportional relationships.

3. **Writing Equations to Represent Relationships** — Let's use equations to solve problems involving proportional relationships.

Section B: Comparing Proportional and Nonproportional Relationships

Section Goals

- Determine whether the values in a table could represent a proportional relationship.
- Use a table to determine whether an equation represents a proportional relationship.

Section Narrative

In this section, students decide whether two quantities are in a proportional relationship. First, students examine tables. They calculate a unit rate for each row of the table and check whether these rates are all the same. If so, the relationship could be proportional. Next, students work with equations. Students use an equation to create a table and determine whether the relationship is proportional. Later, they decide whether a situation involves a proportional relationship based on characteristics of the situation, that is, whether or not it involves a constant rate.

Lessons

1. **Comparing Relationships with Tables** — Let's explore how proportional relationships are different from other relationships.
2. **Comparing Relationships with Equations** — Let's develop methods for deciding if a relationship is proportional.
3. **Solving Problems about Proportional Relationships** — Let's solve problems about proportional relationships.

Section C: Representing Proportional Relationships with Graphs

Section Goals

- Determine whether a given graph represents a proportional relationship.
- Identify the constant of proportionality from the graph of a proportional relationship.
- Interpret points on the graph of a proportional relationship.

Section Narrative

In this section, students work with proportional relationships that are represented with graphs. Students learn that the graph of any proportional relationship lies on a line through the origin that passes through Quadrant I. First, students compare graphs of proportional and nonproportional relationships. Next, they create and interpret graphs of proportional relationships. Later, they compare different proportional relationships that are graphed on the same axes and see that the constant of proportionality indicates the steepness of the line.

Lessons

1. **Graphs of Proportional Relationships** — Let's see how graphs of proportional relationships differ from graphs of other relationships.
2. **Using Graphs to Compare Relationships** — Let's graph more than one relationship on the same grid.
3. **Two Graphs for Each Relationship** — Let's use tables, equations, and graphs to answer questions about proportional relationships.

Section D: Circumference of a Circle

Section Goals

- Recognize that there are proportional relationships between the circumference, diameter, and radius of circles and express these relationships using equations.
- Solve problems involving circumference.

Section Narrative

In this section, students apply proportional reasoning to solve problems involving circumference. The section begins with students measuring squares. Students learn to take into account potential measurement error when deciding whether a list of related measurements could represent a proportional relationship. Next, students investigate circles. They learn the terms radius, diameter, and circumference, as well as a more precise meaning of the term circle. Then, they repeat the process they previously used while measuring squares to measure circles. They recognize that the circumference of a circle is proportional to its diameter. Students estimate the constant of proportionality, and then pi is introduced. Students apply the relationship C = pi d to solve problems. The last lesson of the section focuses on the proportional relationship between the number of times a wheel rotates and the distance it travels. This lesson is optional because it provides more opportunities to practice with proportional relationships that involve circumference.

Lessons

1. **How Well Can You Measure?** — Let's see how accurately we can measure.
2. **Exploring Circles** — Let's explore circles.
3. **Exploring Circumference** — Let's explore the circumference of circles.
4. **Applying Circumference** — Let's use pi to solve problems.

Section E: Area of a Circle

Section Goals

- Justify that the area of a circle can be calculated with the formula A = pi r squared.

Section Narrative

In this section, students explore the area of circular regions. They see an informal derivation that shows where the formula A = pi r squared comes from and then use this formula to solve problems. Students estimate the areas of irregular shapes and then investigate areas of circles, comparing their estimates with the formula. Finally, students find the areas of shapes made up of circles and use circumference and area to solve real-world problems.

Lessons

1. **Estimating Areas** — Let's estimate the areas of weird shapes.
2. **Area of a Circle** — Let's investigate the areas of circles.
3. **Applying Area of Circles** — Let's find the areas of shapes made up of circles.

Section F: Let's Put It to Work

Section Narrative

In this final section, students have the opportunity to apply their thinking from throughout the unit. As this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions. All lessons in this section are optional.

Lessons

1. **Four Representations (Optional)** — Let's contrast relationships that are and are not proportional in four different ways.
2. **Using Water Efficiently** — Let's investigate saving water.
3. **Distinguishing Circumference and Area** — Let's contrast circumference and area.
4. **Stained-Glass Windows (Optional)** — Let's use circumference and area to design stained-glass windows.

Unit 6: Percent Increase and Decrease

Unit Goal

Students deepen their understanding of proportional relationships and percentages, solving multi-step problems and applying percent increase and decrease in real-world contexts.

Unit Narrative

In this unit, students deepen their understanding of proportional relationships and percentages. They solve multi-step problems and work with situations that involve fractional amounts. This builds on the work students did in previous units with ratios, rates, percentages, and proportional relationships. Students will build on this work in high school with exponential functions representing compounded percent increase and decrease. Students begin the unit by revisiting proportional relationships, but this time the given values are fractional amounts. To determine the constant of proportionality, students must compute the quotient of two fractions. Students also make sense of situations where an increase or decrease is expressed as a fraction of the initial amount. They create diagrams and apply the distributive property to generate expressions that represent these situations. They also use long division to write fractions as decimals, including their first introduction to repeating decimals. Next, students make sense of situations where an increase or decrease is expressed as a percentage of the initial amount. They continue creating diagrams and writing equations to represent the situations. They solve for any one of the three quantities—the initial amount, the final amount, or the percentage of the change—given the other two quantities. They also reason about fractional percentages. Then students apply percent increase and decrease to solve problems in a variety of real-world situations, such as tax, tip, interest, markup, discount, depreciation, and commission. Lastly, students make sense of situations where the difference between a correct measurement and an incorrect measurement is expressed as a percentage of the correct amount.

Section A: Proportional Relationships with Fractions

Section Goals

- Create algebraic expressions that represent a situation involving adding or subtracting a fraction of the initial value.
- Solve problems about proportional relationships with fractional quantities.
- Use long division to generate a decimal representation of a fraction.

Section Narrative

In this section, students work with proportional relationships that involve fractional values. First they investigate scaled copies where the side lengths include fractions. Then students solve problems involving constant rates between fractional quantities. They use tables to make sense of proportional situations and solve for unknown values. This section also helps prepare students for learning about percent increase and decrease. Early in the section, students review how to calculate percentages. Later in the section, students examine situations where an increase or decrease is expressed as a fraction of the initial amount. They create diagrams and equations to represent these situations. They use the distributive property to generate equivalent expressions and also use long division to convert fractions to decimals.

Lessons

1. **Revisiting Proportional Relationships** — Let's use constants of proportionality to solve more problems.

2. **More than That, Less than That** — Let's use fractions to describe increases and decreases.
3. **Say It with Decimals** — Let's use decimals to describe increases and decreases.

Section B: Percent Increase and Decrease

Section Goals

- Create algebraic expressions or equations that represent a situation involving percent increase or decrease.
- Use diagrams to solve problems involving percent increase or decrease.

Section Narrative

In this section, students work with percent increase and decrease. First they interpret verbal descriptions and diagrams. They find the new amount after being given the initial amount and the percentage of the increase or decrease. Then students create their own diagrams and solve problems where the final amount is known but the initial amount or the percentage of the change is unknown. Students relate verbal descriptions and algebraic expressions, which is especially useful for solving for an unknown initial amount. In the last lesson of the section, students reason about percentages that are not whole numbers, which prepares them for more real-world applications of percent increase and decrease.

Lessons

1. **Increasing and Decreasing** — Let's use percentages to describe increases and decreases.
2. **One Hundred Percent** — Let's solve more problems about percent increase and percent decrease.
3. **Percent Increase and Decrease with Equations** — Let's use equations to represent increases and decreases.
4. **Part of a Percent** — Let's solve more percent problems with fractional percentages.

Section C: Applying Percentages

Section Goals

- Calculate measurement error, and express it as a percentage of the actual value.
- Solve problems involving tax, tip, simple interest, markup, markdown, or commission.

Section Narrative

In this section, students work with real-world examples of percent increase and decrease. Some applications of percent increase include sales tax, tips, interest, and markup. Applications of percent decrease include discounts, depreciation, and commission. However, it is not necessary for students to memorize the meanings of these terms. The goal is for students to be able to write equations and solve problems without needing to draw a diagram first. Then students work with percent error. They interpret situations where the difference between a correct value and an incorrect value is expressed as a percentage of the correct value. They solve for any one of the three quantities—the correct amount, the incorrect amount, or the percentage of the error—given the other two quantities.

Lessons

1. **Tax and Tip** — Let's learn about sales tax and tips.
2. **Percentage Contexts** — Let's learn about more situations that involve percentages.

3. **Solving Multi-step Percentage Problems** — Let’s solve more problems about sales tax and discounts.
4. **Expressing Error as a Percentage** — Let’s use percentages to describe situations that involve error.

Section D: Let’s Put It to Work

Section Narrative

In this final section, students have the opportunity to apply their thinking from throughout the unit. As this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions. All lessons in this section are optional.

Lessons

1. **Changes on the Earth (Optional)** — Let’s use percentages to make sense of changes on Earth.
2. **Posing Percentage Problems** — Let's explore how percentages are used in the news.

Unit 7: Rational Numbers

Unit Goal

Students learn about negative numbers and ways to represent them on a number line and the coordinate plane, and they perform the four operations on rational numbers.

Unit Narrative

In this unit, students learn about negative numbers and ways to represent them on a number line and the coordinate plane. They perform operations on rational numbers, which are all numbers that can be written as a positive or negative fraction or zero. Students begin by considering situations involving temperature or elevation and interpreting what negative numbers mean in those contexts. Previously, when students worked only with nonnegative numbers, magnitude and order were indistinguishable. In this unit, when comparing two signed numbers, students learn to distinguish between the absolute value of a number (magnitude) and a number's relative position on the number line (order). Then students use tables and number line diagrams to represent changes in temperature or elevation. They extend addition and subtraction from fractions to all rational numbers. And they see that a minus b is equivalent to a plus the opposite of b. Then students use ordered pairs to describe pairs of numbers that include negative numbers. In grade 5, they plotted pairs of positive numbers on the coordinate grid. Here, they plot pairs of rational numbers in all four quadrants of the coordinate plane. They interpret the meanings of plotted points in given contexts and use coordinates to calculate horizontal or vertical distances between two points. Next, students examine multiplication and division. They work with constant velocity, which is a signed number that indicates direction and speed. This allows products of signed numbers to be interpreted in terms of position, direction of movement, and time before or after a specific point. Students use the relationship between multiplication and division to understand how division extends to rational numbers. Then students work with expressions that use the four operations on rational numbers. They also solve problems that involve interpreting negative numbers in context. They solve linear equations of the form $x + p = q$ or $px = q$, where p and q are rational numbers.

Section A: Negative Numbers and Absolute Value

Section Goals

- Interpret a rational number and the absolute value of a number in context.
- Plot rational numbers and their opposites on a number line; know that a number and its opposite have the same absolute value.
- Use words and symbols to compare rational numbers, where a rational number could also be the absolute value of a number.

Section Narrative

Work in this section expands students' understanding of the number system to include both positive and negative numbers. Students begin by examining situations involving temperature and elevation to understand the need for numbers less than 0. They work with both vertical and horizontal number lines to represent positive and negative values, observing the symmetry present in the number line. Students use the number line to compare values and practice ordering rational numbers from least to greatest. They recognize that points to the right on a horizontal number line (or farther up on a vertical number line) have a greater value than points to the left (or farther down). They use the inequality symbols $<$ and $>$ to write comparison statements. Next, students examine more situations that can be represented using both positive and negative numbers, such as money and inventory. The end of this section introduces students to the idea of absolute value, which is the distance of a number from zero. Students

return to working with temperature and elevation. They differentiate between objects' elevations and their distances from sea level.

Lessons

1. **Positive and Negative Numbers** — Let's explore how we represent temperatures and elevations.
2. **Comparing Positive and Negative Numbers** — Let's compare numbers on the number line.
3. **Ordering Rational Numbers** — Let's order rational numbers.
4. **Absolute Value of Numbers** — Let's explore distances from zero more closely.
5. **Comparing Numbers and Distance from Zero** — Let's use absolute value and negative numbers to think about elevation.

Section B: Adding and Subtracting Rational Numbers

Section Goals

- Apply addition and subtraction of signed numbers to represent situations and solve problems.
- Calculate the sum or difference of two rational numbers.

Section Narrative

In this section, students make sense of adding and subtracting signed numbers. They begin by considering how changes in temperature and elevation can be represented with tables, number line diagrams, expressions, and equations. Then they use these examples to generalize rules for adding signed numbers. They apply the rules to signed numbers that represent money contexts, including deposits, withdrawals, and debts. Next, students use the relationship between addition and subtraction to begin subtracting signed numbers. They interpret subtraction problems as addition with an unknown addend and use a number line to find the answer. From this, they generalize that subtraction is the same as adding the opposite. They also learn the convention that "the difference of a and b" means a minus b, and this value can be positive or negative depending on the relative location of a and b on the number line. Lastly, students apply their understanding of adding and subtracting signed numbers to represent situations and solve problems.

Lessons

1. **Changing Temperatures** — Let's add signed numbers.
2. **Changing Elevation** — Let's solve problems about adding signed numbers.
3. **Money and Debts** — Let's apply what we know about signed numbers to money.
4. **Representing Subtraction** — Let's subtract signed numbers.
5. **Finding Differences** — Let's bring addition and subtraction together.

Section C: The Coordinate Plane

Section Goals

- Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
- Use coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

- Working in all four quadrants, plot a point given its coordinates, or identify the coordinates of a given point in the coordinate plane.

Section Narrative

In this section, students explore ways to represent pairs of numbers that include negative values. They do so by extending the horizontal axis to the left of 0 and the vertical axis below 0 and plotting ordered pairs in all four quadrants of the coordinate plane. They reason about how to appropriately scale and label a set of axes in order to most effectively display a given set of coordinate pairs. Students consider situations that include points plotted in all four quadrants and interpret those points in the context of the situation. Students observe patterns between the location of a point and the signs of the numbers in its coordinates. They notice that when the numbers in two ordered pairs differ only by signs, those two points are located on opposite sides of the x- or y-axis. Students also develop strategies for determining the distance between two points on the same horizontal or vertical line. While use of the distance formula or arithmetic with negative numbers is not expected in this course, students may use strategies such as counting grid squares or calculating distances from an axis and adding them together.

Lessons

1. **Constructing the Coordinate Plane** — Let's explore the coordinate plane.
2. **Interpreting Points in a Coordinate Plane** — Let’s examine what points in the coordinate plane can tell us.
3. **Distances in the Coordinate Plane** — Let’s explore distance on the coordinate plane.

Section D: Multiplying and Dividing Rational Numbers

Section Goals

- Apply multiplication and division of signed numbers to represent situations and solve problems.
- Calculate the product or quotient of two rational numbers.

Section Narrative

In this section, students make sense of multiplying and dividing signed numbers. They begin by exploring constant velocity. This builds on their previous work with constant speed but uses signed numbers to represent positions and movement in opposite directions from a reference point. Students calculate the resulting position for a given velocity and time. Then they use these examples to generalize rules for multiplying signed numbers. An optional lesson provides more opportunities for students to build fluency with multiplying rational numbers. Next, students use the relationship between multiplication and division to begin dividing signed numbers. They interpret division problems as multiplication with an unknown factor. From this, they generalize rules for dividing signed numbers. Lastly, students apply their understanding of multiplying and dividing signed numbers to represent situations and solve problems.

Lessons

1. **Multiplying Rational Numbers** — Let's use signed numbers to represent movement.
2. **Multiply!** — Let's get more practice multiplying signed numbers.
3. **Dividing Rational Numbers** — Let's divide signed numbers.
4. **Negative Rates** — Let's apply what we know about signed numbers.

Section E: Four Operations with Rational Numbers

Section Goals

- Apply the four operations with rational numbers to solve problems.
- Solve an equation of the form x + p = q or px = q, where p, q, and x are rational numbers.
- Write an equation of the form x + p = q or px = q (where p, q, and x are rational numbers) to represent a situation.

Section Narrative

In this section, students solve problems involving the four operations with rational numbers. First, they evaluate algebraic expressions involving signed numbers. They extend their understanding of algebraic notation to include negative numbers, by using "next to" notation (that is, numbers or variables placed next to each other, either with or without parentheses) to represent multiplication and the fraction bar to represent division. Next, students examine new contexts that can be represented with rational numbers. They represent situations with equations, solve equations, and interpret their solutions in context of the situation. The focus of this section is representing situations with equations and what it means for a number to be a solution to an equation, rather than methods for solving equations. Such methods are the focus of a later unit.

Lessons

1. **Expressions with Rational Numbers** — Let's develop our signed number sense.
2. **Solving Problems with Rational Numbers** — Let's use all four operations with signed numbers to solve problems.
3. **Solving Equations with Rational Numbers** — Let's solve equations that include negative values.
4. **Representing Contexts with Equations** — Let's write equations to represent situations and solve them.

Section F: Let's Put It to Work

Section Narrative

In this final section, students have the opportunity to apply their thinking from throughout the unit. As this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions. All lessons in this section are optional.

Lessons

1. **Drawing in the Coordinate Plane (Optional)** — Let's draw on the coordinate plane.
2. **The Stock Market** — Let's learn about the stock market.

Unit 8: Data Sets, Distributions, and Sampling

Unit Goal

Students explore key statistical concepts including dot plots, histograms, measures of center and variability, sampling, and probability.

Unit Narrative

This unit is a brief overview of some key statistical concepts. First, students learn about populations and study variables associated with a population. They begin by classifying questions as either statistical or non-statistical—based on whether variable data is necessary to answer the question. This leads to further investigation into variability and data displays, such as dot plots and histograms. As students visualize data, they begin to describe the distribution of data more precisely as they work with mean and mean absolute deviation (MAD). After working with those statistics, students begin to recognize that some distributions are not well-suited to description by mean and MAD. Students are introduced to median, range, and interquartile range as additional measures of center and variability that can be used to describe distributions in some situations. That also leads to the box plot as an additional way to visualize data. Next, students examine different ways to collect data from samples within a population to understand why random selection is useful. Then students generate samples and estimate information about the population from sample data. The unit concludes with an optional section exploring probability. Students are introduced to probability as a way to quantify how likely an event is to happen. They explore the connection between probability and results of repeated experiments, ways to examine the sample space for more complex experiments, and simulating experiments. Note that the introduction of mean absolute deviation is used as an introductory model for understanding variability. Although standard deviation is more mathematically useful, its calculation and meaning may be difficult for students at this level without an understanding of normal distributions.

Section A: Dot Plots and Histograms

Section Goals

- Describe a distribution represented by a dot plot, including informal observations about its center and spread.
- Interpret a histogram to answer statistical questions about a data set.

Section Narrative

In this section, students focus on describing distributions. In particular, they learn to describe the center and spread of a distribution by using informal language to refer to a typical value for a distribution and how spread out the data are. They use dot plots and histograms to represent data, and use the visualization to describe features of a distribution such as clusters, peaks, gaps, and symmetry. Note that in all histograms in this unit, the left-end boundary of each bin or interval is included and the right-end boundary is excluded. For example, the number 5 would not be included in the 0–5 bin, but would be included in the frequency count for the 5–10 bin.

Lessons

1. **Representing Data** — Let's explore different kinds of data.
2. **Using Dot Plots to Answer Statistical Questions** — Let's use dot plots to describe distributions and answer questions.
3. **Interpreting Histograms** — Let's explore how histograms represent data sets.

Section B: Measures of Center and Variability

Section Goals

- Calculate and interpret the mean and mean absolute deviation (MAD) of a data set.
- Calculate and interpret the median and interquartile range (IQR) of a data set.

Section Narrative

In this section, students begin to quantify their understanding of center and spread by finding values for the mean and mean absolute deviation (MAD). The mean is explained as a way of fairly sharing as well as a balance point to give additional intuition into the measure of center. Then, students see that, even with the same mean, distributions can be very different and that a description of a measure of variability is often important. They use mean absolute deviation as a way to describe the variability of a distribution in a way that has some meaning. Next, students add “median,” “range,” and “interquartile range” to their methods of describing a measure of center or measure of variability. They use the symmetry of a distribution to determine whether mean or median is likely to be a better description of the center. Then they explore box plots as a way to visualize a summary of data using the five-number summary including the minimum, maximum, median, and 2 other quartiles.

Lessons

1. **The Mean** — Let’s explore the mean of a data set and what it tells us.
2. **Variability and MAD** — Let’s use mean and MAD to describe and compare distributions.
3. **The Median** — Let's explore the median of a data set and what it tells us.
4. **Box Plots and Interquartile Range** — Let's explore how box plots can help us summarize distributions.

Section C: Sampling

Section Goals

- Describe methods to obtain a random sample from a population, and explain why it is representative of the population.
- Explain why samples are necessary and describe a sample and population for a given statistical question.
- Use the mean of a random sample to make inferences about the population.

Section Narrative

In this section, students consider much larger populations to motivate the need to sample to obtain data. This leads to considering how some samples may be more representative of the population than others and the idea that random selection is more likely to produce representative samples. Then students use samples to gain information about the populations they represent. In particular, students estimate measures of center for populations based on information from a sample.

Lessons

1. **Larger Populations** — Let’s compare larger groups.
2. **What Makes a Good Sample?** — Let’s see what makes a good sample.
3. **Sampling in a Fair Way** — Let’s explore ways to get representative samples.
4. **Estimating Population Measures of Center** — Let’s use samples to estimate measures of center for the population.

5. **More about Sampling Variability (Optional)** — Let's compare samples from the same population.

Section D: Probability

Section Goals

- Describe a multi-step experiment that could be used to simulate a compound event in a real-world situation, and justify that it represents the situation.
- Interpret or create a list, table, or tree diagram that represents the sample space of a compound event, and use the sample space to write the probability for an event.
- Use the results from a repeated experiment to estimate the probability of an event, and justify the estimate.
- Use the sample space to determine the probability of an event, and express it as a fraction, decimal, or percentage.

Section Narrative

This section is optional because it gives a light introduction to probability, which is not essential for the unit's main focus on statistics. In this section, students learn how to quantify the likelihood of events using probability. First, they list the sample space for a chance experiment and use it to assign values to the likelihood, such as 50% or one-half. Then students recognize these values as the fraction of times an event is likely to happen after many repeated trials of the chance experiment. Next, students expand their understanding of probability to more complex chance experiments that involve multiple steps, such as rolling a number cube and flipping a coin. They use structures such as tree diagrams, tables, and lists to record the sample spaces. After using those sample spaces to write probabilities, they design their own simulation to estimate the probability of an experiment that would be difficult to repeat.

Lessons

1. **What Are Probabilities?** — Let's find out what's possible.
2. **Estimating Probabilities through Repeated Experiments** — Let's do some experimenting.
3. **Keeping Track of All Possible Outcomes** — Let's explore sample spaces for experiments with multiple parts.
4. **Multi-step Experiments** — Let's look at probabilities of experiments that have multiple steps.
5. **Designing Simulations** — Let's simulate some real-life scenarios.

Unit 9: Putting It All Together

Unit Goal

In this optional unit, students use concepts and skills from previous units to solve problems involving Fermi problems, voting systems, and designing a course.

Unit Narrative

In this optional unit, students use concepts and skills from previous units to solve problems. The first section explores a variety of different contexts, such as population density, Fermi problems, measurement error, and energy usage. The second section has five lessons about different systems of voting. In the last section, students build a trundle wheel and design a five-kilometer race course. All related standards in this unit have been addressed in prior units. These sections provide an optional opportunity for students to go more deeply and make connections between domains.

Section A: Making Connections

Section Goals

- Apply proportional reasoning, algebraic expressions, and percentages to model real-world situations.
- Choose an appropriate level of accuracy when reporting estimates of quantities.
- Estimate quantities in a real-world situation, and explain the estimation strategy.
- Interpret and create diagrams involving a rectangle decomposed into squares.

Section Narrative

These optional lessons do not build on each other and can be completed in any order. In this section, students apply proportional reasoning, fractions, and percentages to make sense of situations and solve problems. Each lesson builds on skills developed in earlier units. All standards in this section have been addressed in prior units. This section provides an optional opportunity for students to go more deeply and make connections between domains.

Lessons

1. **Fermi Problems (Optional)** — Let’s make some estimates.
2. **Energy Flow (Optional)** — Let's compare numbers and rates.
3. **Making Paper (Optional)** — Let's decide what material to make paper from.
4. **If Our Class Were the World (Optional)** — Let’s use math to better understand our world.
5. **Rectangle Madness (Optional)** — Let’s cut up rectangles.
6. **Rectangle Fractions (Optional)** — Let's compare fractions and rectangles.
7. **Measurement Error (Optional)** — Let’s check how accurate our calculations are.
8. **How Crowded Is This Neighborhood? (Optional)** — Let’s see how proportional relationships apply to where people live.

Section B: Voting

Section Goals

- Choose and justify which voting system seems the fairest for dealing with more than two choices.

- Critique a statement reporting the results of a vote.
- Interpret voting situations involving more than two choices through the use of ratios and percentages.

Section Narrative

This optional sequence of lessons explores the mathematics of voting. These lessons build on one another and should be completed in order. It is not necessary to do all five lessons to get some benefit, although more connections are made the further one gets. Students begin by comparing election results to determine which group is more in favor of the result. They then explore supermajorities and see the impact of elections with few voters participating. Students compare how the results of the same election can change depending on the system of election that is used. Specifically, they look at plurality, runoff, and instant runoff elections. Finally, students try to fairly assign representatives and explore how the design of districts can affect election results. All of the lessons in this section are important for citizens in a democracy to understand. Teachers may wish to collaborate with a civics or government teacher to learn how the fictional middle-school situations in these lessons relate to real-world elections.

Lessons

1. **How Do We Choose? (Optional)** — Let's vote and choose a winner!
2. **More than Two Choices (Optional)** — Let's explore different ways to determine a winner.
3. **Comparing Voting Systems (Optional)** — Let's compare voting systems.
4. **Picking Representatives (Optional)** — Let's think about fair representation.
5. **Designing Districts (Optional)** — Let's design districts.

Section C: Designing a Course

Section Goals

- Calculate the distance of a path using the circumference and number of rotations of a trundle wheel.
- Compare and contrast methods for measuring long distances.

Section Narrative

This optional sequence of lessons can be done anytime after Unit 4 is completed. In this section, students explore different ways to measure long distances and then design a five-kilometer course for their school. They begin by measuring a smaller track using a variety of tools and strategies. Then they are introduced to a trundle wheel, a tool for measuring distance on uneven or curving surfaces. Students build their own trundle wheels and compare this method of measuring with their earlier choices. Finally, students use a map of the school grounds to plan a five-kilometer course and use their trundle wheels to verify its length. The lessons in this section build on each other and should be completed in order. All standards in this section have been addressed in prior units. This section provides an optional opportunity for students to go more deeply and make connections between domains.

Lessons

1. **Measuring Long Distances over Uneven Terrain (Optional)** — Let’s measure long distances over uneven terrain.
2. **Building a Trundle Wheel (Optional)** — Let’s build a trundle wheel.
3. **Using a Trundle Wheel to Measure Distances (Optional)** — Let’s use our trundle wheels.

4. **Designing a 5K Course (Optional)** — Let's map out the 5K course.

About This Syllabus

This syllabus was compiled from the publicly available Illustrative Mathematics 6-8 Accelerated v.360 curriculum at accessim.org. Unit and section narratives, goals, and lesson titles are sourced from the teacher view. Optional lessons are labeled “(Optional).”
Assessments (checkpoints, mid-unit, and end-of-unit assessments) are referenced in the source curriculum but require a Kendall Hunt educator account to view; they are not included here.
https://accessim.org/6-8-accelerated/accelerated-6?a=teacher