Algebra 2

Full Syllabus

Illustrative Mathematics® 9-12 AGA (v.360)

8 Units · Unit Goals · Section Goals · Lesson-by-Lesson Outline

Source: https://accessim.org/9-12-aga/algebra-2?a=teacher

Course Overview

Algebra 2 of the Illustrative Mathematics 9-12 AGA curriculum is organized into 8 units. Students build on their prior work with linear, quadratic, and exponential functions to explore polynomial and rational functions; extend exponent rules to rational exponents and introduce complex numbers; develop logarithmic functions; study transformations of functions; build trigonometric functions from the unit circle; and engage with statistical inference using study design, normal distributions, sampling variability, and experimental analysis.

The unit sequence is:

- Unit 1: Sequences and Functions
- Unit 2: Polynomial Functions
- Unit 3: Rational Functions and Equations
- Unit 4: Complex Numbers and Rational Exponents
- Unit 5: Exponential Functions and Equations
- Unit 6: Transformations of Functions
- Unit 7: Trigonometric Functions
- Unit 8: Statistical Inferences

Each unit is divided into lettered sections (A, B, C, D, E as applicable), and each section is a sequence of daily lessons. Optional lessons are noted with “(Optional).”

Unit 1: Sequences and Functions

Unit Goal

Students revisit functions through sequences, seeing arithmetic and geometric sequences as linear and exponential functions restricted to integer domains, and represent and model situations using sequences.

Unit Narrative

This unit provides students an opportunity to revisit functions by way of sequences. Through many concrete examples, students will see arithmetic and geometric sequences as linear and exponential functions restricted to a domain that is a subset of the integers. This unit reinforces understanding of functions by using multiple representations (including graphs, tables, and expressions). Students begin with an invitation to describe sequences with informal language and learn to identify if a sequence is geometric, arithmetic, or neither. Students write out the terms of sequences arising from mathematical situations, in addition to interpreting and creating tables and graphs about the given relationship. Students learn that sequences are a type of function in which the input variable is the position and the output variable is the term at that position. They learn to interpret sequences and then use function notation to write their own recursive definitions of the sequences. Students make connections between the sequences and different representations of functions. Students then build on their prior knowledge of linear and exponential functions to develop explicit formulas for arithmetic and geometric sequences and to model several situations represented in different ways. In the last section of the unit, students use sequences to model several situations represented in different ways. This work is meant to touch on some practices that must be attended to while modeling, such as choosing a good model and identifying an appropriate domain. Students also recognize that a sequence is an appropriate type of function to use as a model for these situations since the domain of each situation is a subset of the integers. Finally, students encounter some situations in which it makes sense to compute the sum of a finite sequence.

Section A: Sequences

Section Goals

- Determine missing terms in arithmetic and geometric sequences.
- Determine the growth factor of a geometric sequence and the rate of change of an arithmetic sequence.

Section Narrative

In this section, students learn about sequences. Focusing on geometric and arithmetic sequences, students determine missing values, calculate growth factors and rates of change, and represent sequences using tables and graphs. The last lesson of this section is optional. It provides activities to teach students how to use spreadsheets and graphing technology, which are useful for working with sequences and give students more options for choosing appropriate tools strategically in their future work. A note on language and notation: In earlier courses, students may have learned that a ratio is an association between two or more quantities. In more advanced work, such as this course, “ratio” is typically used as a synonym for “quotient.” This expanded use of the word “ratio” comes into play in this section with the introduction of the term “common ratio.”

Lessons

1. **A Towering Sequence** — Let’s explore the Tower of Hanoi.
2. **Introducing Geometric Sequences** — Let’s explore growing and shrinking patterns.

3. **Different Types of Sequences** — Let’s look at other types of sequences.
4. **Using Technology to Work with Sequences (Optional)** — Let’s use technology to create a sequence.

Section B: Representing Sequences

Section Goals

- Create a table, graph, or recursive definition of a sequence from given information.

Section Narrative

In this section, students learn that sequences are functions and how to use function notation to define sequences recursively. To strengthen their understanding of recursive definitions, students practice representing sequences in familiar ways with tables and graphs. Throughout this section, students exchange questions, ideas, and strategies with their classmates. Regarding the notation students use when writing an equation that defines a sequence, avoid being overly prescriptive. It is more important that students can describe the rule correctly than it is that they can do it in a particular format.

Lessons

1. **Sequences Are Functions** — Let's learn how to define a sequence recursively.
2. **Representing Sequences** — Let’s look at different ways to represent a sequence.
3. **Representing More Sequences** — Let's learn about Info Gaps.

Section C: What’s the Equation?

Section Goals

- Create equations for sequences representing situations.
- Identify restrictions on the domain of a sequence based on the context.

Section Narrative

In the final section, students study the domain of a function, explore types of equations, and write equations to define sequences given tables, visual patterns, and situations. Students begin by moving from recursive to explicit definitions of sequences and continue making connections between sequences and functions. Students also explore how changing the domain can change the equation for the sequence, and they consider reasonable domains for given contexts. Students also consider how the choice between recursive and explicit equations depends on what they are trying to do and that some representations are more efficient than others for achieving their goals. Next, students practice writing equations for situations, both real-world and abstract, that can be represented by sequences, and they use the models they create to answer questions. Finally, students apply their knowledge of sequences to several situations in which finding the sum makes sense.

Lessons

1. **The nth Term** — Let’s define sequences.
2. **Situations and Sequence Types** — Let’s decide what type of sequence we are looking at and how to represent it.
3. **Adding Up** — Let’s look at sequences and the sum of their terms.

Unit 2: Polynomial Functions

Unit Goal

Students extend their work with linear and quadratic functions to polynomials of higher degree, moving flexibly between standard and factored forms, sketching graphs from key features (zeros, multiplicity, end behavior), and dividing polynomials to apply the Remainder Theorem.

Unit Narrative

This unit extends students' previous work with linear and quadratic functions as they investigate polynomials of higher degree. Students rewrite polynomials in different forms, recognizing the benefits of the various forms for their ability to reveal the structure of key features of their graphs. The unit begins with an introduction to two situations that can be modeled by a polynomial function. Students build their understanding of what polynomials are and what their graphs can look like. Certain aspects, such as end behavior, will be important in a later unit when students explore the end behavior of rational functions. Focusing on functions expressed in factored form and their graphs, students connect that a factor means a zero of the function and a horizontal intercept. The effect of the degree and leading coefficient on end behavior is established along with the effect of multiplicity on the shape of the graph near zeros of the function. Taking in all of these features, students learn to sketch polynomial functions expressed as a product of linear factors. In a previous course, students used the distributive property to multiply factors, and also factored quadratics. Opportunities to review these skills and apply them to polynomials of higher degree are embedded throughout the unit and in practice problems. This prepares students for the final section where students divide a polynomial written in standard form by a suspected factor. From there, the connection between division and multiplication equations is used to establish the Remainder Theorem. This allows the conclusion that if a polynomial has a zero at a value, then it must also have the corresponding linear expression as a factor.

Section A: What Is a Polynomial?

Section Goals

- Generalize that when polynomials are combined by addition, subtraction, or multiplication, the result is a polynomial.
- Identify and interpret relative minimums, relative maximums, the degree, and the constant term of polynomials and their graphs.

Section Narrative

This section introduces students to polynomials in different forms and connects features of the equations of polynomials to their graphs. Students begin by exploring two contexts that can be modeled by polynomials. First students cut out identical squares from each corner of a rectangular sheet of paper and then fold it to create an open-top box. The volume of the box is a function of the side length of the square cutouts and is most easily expressed by a cubic function written in factored form. Next students consider an investment that earns interest each year and is most easily modeled by a polynomial written in standard form. These two examples showcase how the form of a polynomial can make certain features more clear. Students continue to explore polynomials written in standard form by first matching equations to graphs and then using technology to explore how to write equations that have specific features in their graphs. Finally, students consider similarities between arithmetic operations with integers and arithmetic operations with polynomials. They generalize that, as with integers, addition, subtraction, and multiplication of two polynomials results in another polynomial. This is not always the case when dividing two polynomials.

Lessons

1. **Let’s Make a Box** — Let’s investigate volumes of different boxes.
2. **Funding the Future** — Let’s investigate an investment situation that can be modeled with a function.
3. **Introducing Polynomials** — Let’s see what polynomials can look like.
4. **Combining Polynomials** — Let's do arithmetic with polynomials.

Section B: Working with Polynomials

Section Goals

- Determine properties of polynomials by representing them in standard or factored form.
- Generate a possible expression for a polynomial function given the horizontal intercepts of the function.
- Identify zeros of polynomial functions written in factored form.

Section Narrative

In this section, students see how the standard and factored forms of a polynomial each highlight different features, and they make use of structure to move flexibly between them. Students begin by noticing patterns between the factors of a polynomial and the x-intercepts of the graph of the polynomial. They extend their previous work with quadratics to generalize that if factors multiplied together equal 0 for a specific value of x, then at least one of the factors must also equal 0 at that value of x. Next students see that while factored form is useful for finding zeros and x-intercepts, standard form is more useful for determining the degree and constant term of a polynomial. They build on their knowledge of the distributive property to rewrite polynomials from factored to standard form. Finally, students consider the effect of a constant factor on the graph of the polynomial. They write possible equations for polynomials with specified horizontal intercepts, and consider appropriate viewing windows when using technology to graph polynomials.

Lessons

1. **Connecting Factors and Zeros** — Let’s investigate polynomials written in factored form.
2. **Different Forms** — Let’s use the different forms of polynomials to learn about them.
3. **Using Factors and Zeros** — Let’s write some polynomials.

Section C: Graphs of Polynomials

Section Goals

- Calculate the solution to a system of polynomial equations.
- Create equations for polynomial functions with specific end behavior.
- Use zeros and multiplicities to sketch a graph of a polynomial function given in factored form.

Section Narrative

In this section, students use characteristics of polynomial functions such as factors, degree, and multiplicity to sketch graphs and describe end behavior. They begin by noticing patterns between the degree of a polynomial and the shape of its graphs, learning precise language for describing what happens to the function as the inputs get larger and larger in either the positive or negative direction. Students continue to notice patterns as they explore how the sign of the leading coefficient and the multiplicity of factors affect the graph of the polynomial. Students put all of these considerations together as they sketch graphs of polynomials written in factored form. Lastly, students build on previous work with systems of linear equations to solve systems of equations that include at least one quadratic equation. They use both graphing and algebraic strategies to find points of intersection for two polynomial functions.

Lessons

1. **End Behavior (Part 1)** — Let’s investigate the shape of polynomials.
2. **End Behavior (Part 2)** — Let’s describe the end behavior of polynomials.
3. **Multiplicity** — Let’s sketch some polynomial functions.
4. **Finding Intersections** — Let’s think about two polynomials at once.

Section D: Polynomial Division

Section Goals

- Generalize that for a polynomial and a number c, the remainder on division by (x − c) is the value of the polynomial at c.
- Rewrite a polynomial as the product of linear factors.

Section Narrative

In this section students are introduced to polynomial division. They begin by using diagrams to help reason about the distributive property. Next they explore parallels between long division with integers and long division with polynomials. They compare using the algorithm for long division with the reasoning used in completing a diagram. Next, using the Information Gap structure, students put together everything they have learned about polynomials to sketch a graph of a polynomial written in standard form, given one linear factor. They must first use division to rewrite the polynomial as the product of linear factors, and then take into account the degree, zeros, and multiplicity of each factor in order to sketch a graph of the polynomial, including end behavior. Students continue making connections as they compare the meaning of the remainder when dividing integers and the meaning of the remainder when dividing polynomials. They use the fact that if a linear expression is a factor of a polynomial, then the remainder on division is zero. Students are also briefly introduced to the Remainder Theorem that states: When a polynomial is divided by (x − c), then the remainder is equal to the value of the polynomial at c.

Lessons

1. **Polynomial Division (Part 1)** — Let’s learn a way to divide polynomials.
2. **Polynomial Division (Part 2)** — Let’s learn a different way to divide polynomials.
3. **What Do You Know about Polynomials?** — Let's put together what we've learned about polynomials so far.
4. **The Remainder Theorem** — Let's connect division and the Remainder Theorem.

Unit 3: Rational Functions and Equations

Unit Goal

Students transition from polynomials to rational functions and equations, analyzing asymptotic behavior, solving rational equations (including identifying extraneous solutions), proving polynomial identities, and deriving the finite geometric series sum formula.

Unit Narrative

Building on the “Polynomial Functions” unit, in this unit, students transition to working with rational functions, rational equations, and identities. The unit begins with an introduction to rational functions as students consider situations they can model, such as when minimizing surface area or determining average costs. Students examine the asymptotic behavior of their graphs, which relates to the structure of the equations. Students continue to build on structure as they use polynomial division to rewrite rational expressions for the purpose of identifying the end behavior of the function. Students then focus on solving rational equations and making sense of how some methods can lead to possible solutions that are in fact not solutions (so-called “extraneous solutions”). In the final section, students study identities. Students sharpen their skills manipulating expressions while proving, or disproving, that two expressions are equivalent. The unit concludes with a return to geometric sequences first examined in the previous unit and, using a polynomial identity proved at the start of the section, students derive the formula for the sum of a finite geometric series before using the formula to solve problems.

Section A: Rational Functions

Section Goals

- Identify asymptotes of rational functions from their equations.
- Interpret rational functions in context.

Section Narrative

This section introduces students to rational functions by connecting features of the functions’ equations and graphs, such as vertical and horizontal asymptotes. The first lesson purposely echoes the first lesson of the “Polynomial Functions” unit by focusing on a geometric concept. This encourages comparing rational and polynomial functions during the lesson, before revealing at the end that polynomials are a specific type of rational function. Throughout this section students work with both real-world and abstract contexts as they make connections between different representations of rational functions. Students make sense of the behavior of rational functions near vertical asymptotes. Their understanding of end behavior evolves with the introduction of horizontal asymptotes. The final lesson of the section reintroduces the use of polynomial long division to see the structure of an equation in a different way, revealing relationships between the end behavior and the degrees of the numerator and denominator of a rational equation.

Lessons

1. **Minimizing Surface Area** — Let’s investigate surface areas of different cylinders.
2. **Graphs of Rational Functions (Part 1)** — Let’s explore graphs and equations of rational functions.
3. **Graphs of Rational Functions (Part 2)** — Let’s learn about horizontal asymptotes.
4. **End Behavior of Rational Functions** — Let’s explore the end behavior of rational functions.

Section B: Rational Equations

Section Goals

- Create simple equations involving a rational expression to solve problems.
- Solve equations involving rational expressions, and identify any extraneous solutions.

Section Narrative

The focus of this section is on solving rational equations. Students begin by writing and solving equations involving a rational expression that is equal to a rational number. As the lessons progress, students will solve increasingly challenging rational equations, with the last lesson focusing on understanding how extraneous solutions may arise.

Lessons

1. **Rational Equations (Part 1)** — Let’s write and solve some rational equations.
2. **Rational Equations (Part 2)** — Let’s write and solve some more rational equations.
3. **Solving Rational Equations** — Let’s think about how to solve rational equations strategically.

Section C: Polynomial Identities

Section Goals

- Prove identities.
- Use the formula for the sum of the first n terms in a geometric sequence to solve problems.

Section Narrative

The goal of this section is for students to build their skills working with larger, more complex equations. The first two lessons focus on identities. Students learn what identities are and how to determine if an equation is an identity. During these lessons, students show that a key polynomial identity holds. This equation gives the foundation for the formula for the sum of a finite geometric series, which is the focus of the latter two lessons. Students practice applying the formula to both abstract and real world situations. A note on language: Since series are beyond the scope of this course, these materials do not introduce the term at this time. Student-facing materials refer to “the sum of a finite geometric sequence” instead, linking back to earlier work with sequences.

Lessons

1. **Polynomial Identities (Part 1)** — Let’s learn about polynomial identities.
2. **Polynomial Identities (Part 2)** — Let’s explore some other identities.
3. **Summing Up** — Let’s figure out a better way to add numbers.
4. **Using the Sum** — Let’s calculate some totals.

Unit 4: Complex Numbers and Rational Exponents

Unit Goal

Students extend exponent rules to rational exponents, solve equations involving square and cube roots, develop the concept of complex numbers, and solve quadratic equations with real or complex solutions.

Unit Narrative

In this unit, students use what they know about exponents and radicals to extend exponent rules to include rational exponents, solve equations involving squares and square roots, develop an understanding of complex numbers, and solve quadratic equations that include complex roots. The unit opens with an optional review of exponent rules before using those rules to justify rational exponents in terms of radicals. The new rule leads to an exploration of square and cube roots as solutions to equations. In particular, students learn that positive numbers have two square roots and that the radical symbol represents only the positive root. Students explore equations involving square and cube roots to recognize that squaring each side of an equation can introduce solutions that are not solutions to the original equation. The third section introduces imaginary and complex numbers by proposing i as a solution to the equation x squared equals negative one. Students explore the implications of this new type of number by representing it on an imaginary axis off of the real number line. This exploration includes adding imaginary and real numbers together to get complex numbers, and then adding and multiplying complex numbers. Next, students use their new understanding of complex numbers to solve more quadratic equations. Finally, there is an optional exploration of data representation on a map using circles with area proportional to the data, in which students must use their understanding of square roots to find the radii of the appropriate circles.

Section A: Exponent Properties

Section Goals

- Use both radicals and exponents to represent numbers.

Section Narrative

In this section, students use their understanding of exponent rules to interpret rational exponents such as x to the one-half as the square root of x for positive numbers x. The first two lessons serve as optional review of exponent rules and the geometric meaning of square and cube roots. If students demonstrate an understanding of these concepts, the lessons can be safely skipped. Then students look at expressions involving unit fractions in an exponent and use exponent rules to recognize the relationship between rational exponents and radicals. The understanding is expanded to other positive rational exponents and then to any rational exponent.

Lessons

1. **Properties of Exponents (Optional)** — Let’s use integer exponents.
2. **Square Roots and Cube Roots (Optional)** — Let’s think about square and cube roots.
3. **Exponents That Are Unit Fractions** — Let’s explore exponents like one-half and one-third.
4. **Positive Rational Exponents** — Let’s use roots to write exponents that are fractions.
5. **Negative Rational Exponents** — Let’s investigate negative exponents.

Section B: Solving Equations with Square and Cube Roots

Section Goals

- Calculate solutions to radical equations.
- Explain why a radical equation has 0 solutions or 1 solution.

Section Narrative

In this section, students develop a familiarity with equations involving square and cube roots. They begin the section by recognizing that positive numbers have two square roots and recognize the benefit of understanding the radical symbol to represent the positive square root. The restriction leads to a need for caution, though, when students see that solving equations involving square roots can sometimes lead to equations with solutions that are not solutions to the original equation. After recognizing that this caution is not necessary for cube roots, students solve equations including square and cube roots. This section ends with an optional lesson that can be used for extra practice. Note that students have not yet been introduced to imaginary numbers, so throughout this section, the materials say that equations such as x squared equals a negative number have no solution. After students encounter imaginary and complex numbers later in the unit, a distinction will be made about real solutions.

Lessons

1. **Squares and Square Roots** — Let's compare equations with squares and square roots.
2. **Inequivalent Equations?** — Let's see what happens when we square each side of an equation.
3. **Cubes and Cube Roots** — Let's compare equations with cubes and cube roots.
4. **Solving Radical Equations (Optional)** — Let's practice solving radical equations.

Section C: A New Kind of Number

Section Goals

- Add, subtract, and multiply complex numbers, and represent the solutions in the form a + bi.

Section Narrative

In this section, students are introduced to imaginary and complex numbers. After being reminded that there was a time when negative numbers were new to them and it took a while to understand how to use them, students are told that i is another new kind of number. Just as the positive number line was extended to show negative values, the real number line is extended to create a complex plane to visualize imaginary and complex numbers. Students then learn how to add and multiply complex numbers. An optional lesson is an opportunity to go beyond the standards to see how increasing powers of complex numbers follow a pattern. It is typical to write complex numbers in the form a + bi with real numbers a and b. This convention is generally followed in the materials for this section.

Lessons

1. **A New Kind of Number** — Let's invent a new number.
2. **Introducing the Number i** — Let's meet i.
3. **Arithmetic with Complex Numbers** — Let's work with complex numbers.
4. **Multiplying Complex Numbers** — Let's multiply complex numbers.
5. **More Arithmetic with Complex Numbers (Optional)** — Let's practice adding, subtracting, and multiplying complex numbers.
6. **Working Backward** — Let's use what we've learned about multiplying complex numbers.

Section D: Solving Quadratics with Complex Numbers

Section Goals

- Create quadratic equations that have either real or non-real solutions.
- Solve quadratic equations, and explain the solution method.

Section Narrative

In this section, students return to solving quadratic equations. This time, they are able to find any real or complex solutions. Students use reasoning, completing the square, and the quadratic formula to solve the equations. Students use the structure of the quadratic formula to help determine the number and type of solutions. Although the discriminant is mentioned, students should not be asked to memorize the expression separately from the quadratic formula. They should be able to reason about the term in the quadratic formula to determine whether solutions are real or complex. The first lesson of the section is optional because it revisits how to complete the square and use the quadratic formula to solve quadratic equations. If students are fluent with this strategy, the lesson can be safely skipped. The last lesson of the section is optional practice in which students create their own quadratic equations so that they have the required number and type of solutions.

Lessons

1. **Solving Quadratics (Optional)** — Let’s solve quadratic equations.
2. **Completing the Square and Complex Solutions** — Let’s find complex solutions to quadratic equations by completing the square.
3. **The Quadratic Formula and Complex Solutions** — Let’s use the quadratic formula to find complex solutions to quadratic equations.
4. **Real and Non-Real Solutions (Optional)** — Let’s create quadratic equations with specific types of solutions.

Section E: Let's Put It to Work

Section Narrative

In this final section, students have the opportunity to apply their thinking from throughout the unit. As this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions.

Lessons

1. **Drawing Proportional Circles (Optional)** — Let's draw a map of data.

Unit 5: Exponential Functions and Equations

Unit Goal

Students extend exponential functions to rational and continuous inputs, introduce logarithms as a way to solve exponential equations, study the constant e, and use exponential and logarithmic functions and their graphs to model and solve problems.

Unit Narrative

In this unit, students explore exponential and logarithmic functions. The unit begins with students recalling geometric sequences and drawing a connection to the growth or decay of values by a constant growth factor. This leads to expressing exponential relationships using functions of the form $f(x) = a \cdot b^x$, where a is the value of the function when x is 0 and b is the growth factor. Students use different rational inputs, including negative values and values between integers, to better understand exponential functions and their meaning in various contexts. This includes an exploration of growth factors over intervals of different lengths. For example, the same population growth can be described using a growth factor per decade or a different growth factor per year. The exploration of variables used as an exponent leads to a need to solve equations for the variable and to the introduction of logarithms. Students are presented with traditional logarithm patterns and asked to find patterns and relationships, which leads them to discover that logarithms represent a way to rewrite exponential equations. The constant e is introduced and students compare functions of the form $a \cdot b^x$ with functions of the form $a \cdot e^{(kx)}$. Finally, students solve exponential and logarithmic equations using graphical and algebraic methods and then interpret the solutions in context. The logarithms in this unit are primarily focused on the bases 10, 2, and e, although other positive bases are mentioned. Note that, throughout the unit, the bases for both exponential expressions and logarithms are assumed to be positive.

Section A: Growing and Shrinking

Section Goals

- Interpret the meaning of a rational input to an exponential function in context.
- Understand that an exponential function changes by a constant factor over equal intervals.

Section Narrative

In this section, students revisit geometric sequences and growth and decay by a fixed percentage to begin thinking about exponential functions. Then students make mathematical and contextual sense of different types of rational inputs, such as negative values or values between integers. Students recognize that exponential functions grow or decay by the same multiple over intervals of equal length. Then they apply that understanding to recognize that growth over subintervals of a certain length must grow or decay by a corresponding factor. Students convert the units of the input, like converting a function written for inputs representing decades to inputs representing years, using this understanding of the growth factor. An optional lesson is available early in the unit for additional review connecting different representations of exponential functions, such as verbal descriptions, tables, graphs, and equations. Another optional lesson is offered later in the unit for students to practice using their understanding of exponential decay with inputs that are not whole numbers.

Lessons

1. **Growing and Shrinking** — Let’s calculate exponential change.
2. **Representations of Growth and Decay (Optional)** — Let’s revisit ways to represent exponential change.

3. **Understanding Rational Inputs** — Let's look at exponential functions where the input values are not whole numbers.
4. **Representing Functions at Rational Inputs** — Let's find how quantities are growing or decaying over fractional intervals of time.
5. **Changes Over Rational Intervals** — Let's look at how an exponential function changes when the input changes by a fractional amount.
6. **Writing Equations for Exponential Functions** — Let's decide what information we need to write an equation for an exponential function.
7. **Interpreting and Using Exponential Functions (Optional)** — Let's explore the ages of ancient things.

Section B: Missing Exponents

Section Goals

- Create equivalent equations in exponential and logarithmic forms.
- Estimate the value of a logarithmic expression.

Section Narrative

In this section, students are introduced to logarithms by first recognizing a need to find a way to get a value for an unknown exponent, then examining the structure of a log table to find the connection between exponents and the values in the table. Logarithms are first introduced using base 10 because they are commonly used and seeing the connection to the number of digits in a number is more easily seen. Then students are introduced to logarithms with a base of 2 before extending their understanding to other bases. Students learn how to use their calculators to compute values for logarithms in base 10, but are only required to know logarithms of other bases when the solution is an integer. For other values, students are only asked to estimate the value between consecutive integers.

Lessons

1. **Unknown Exponents** — Let's find unknown exponents.
2. **What Is a Logarithm?** — Let's learn about logarithms.
3. **Interpreting and Writing Logarithmic Equations** — Let's look at logarithms with different bases.
4. **Evaluating Logarithmic Expressions** — Let's find some logs!

Section C: The Constant e

Section Goals

- Interpret the parameters in the equations of exponential functions with the form $a \cdot e^{(kt)}$.

Section Narrative

This section introduces the constant e as a value often used as a base for exponential functions exhibiting continuous growth. First, they look for a connection between e and the expression $(1 + 1/n)^n$. Then, students look at contexts comparing continuous growth rates to discrete growth rates before turning to the natural logarithm as a way to solve equations stemming from these contexts.

Lessons

1. **The Number e** — Let's learn about the number e.

2. **Exponential Functions with Base e** — Let’s look at situations that can be modeled using exponential functions with base e.
3. **Solving Exponential Equations** — Let’s solve equations using logarithms.

Section D: Logarithmic Functions and Graphs

Section Goals

- Use graphs and logarithms to calculate or estimate solutions to exponential equations.

Section Narrative

In this section, students use graphs to better understand logarithmic functions. They use graphs to approximate solutions to exponential and logarithmic equations and then interpret those solutions in context. They also compare the graphs of logarithmic functions that have different bases.

Lessons

1. **Using Graphs and Logarithms to Solve Problems (Part 1)** — Let’s use graphs and logarithms to solve problems.
2. **Using Graphs and Logarithms to Solve Problems (Part 2)** — Let’s compare exponential functions by studying their graphs.
3. **Logarithmic Functions** — Let’s graph log functions.

Section E: Let's Put It to Work

Section Narrative

In this final section, students have the opportunity to apply their thinking from throughout the unit. As this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions.

Lessons

1. **Applications of Logarithmic Functions (Optional)** — Let’s measure acidity levels and earthquake strengths.

Unit 6: Transformations of Functions

Unit Goal

Students examine how functions can be transformed by translations, reflections, and scaling of inputs and outputs, generalize even and odd symmetry, and apply transformations to model real-world data.

Unit Narrative

In this unit, students consider functions as a whole and understand how they can be transformed to fit the needs of a situation, which is an aspect of modeling with mathematics. Prior to this unit, students have worked with a variety of function types, such as polynomial, radical, and exponential. Students will build on this work in a later unit to transform periodic functions. By saving the introduction of trigonometric functions until after a study of transformations, students have the opportunity to revisit transformations from a new perspective, which reinforces the idea that all functions, even periodic ones, behave the same way with respect to translations, reflections, and scale factors. The unit begins with students informally describing transformations of graphs, eliciting their prior knowledge, and establishing language that will be refined throughout the unit. Students begin their investigation with vertical and horizontal translations and reflections across the axes. This leads to both algebraic and geometric descriptions of a function as even, odd, or neither. Students continue to deepen their understanding by exploring the effects of multiplying the input and output of a function by a scale factor. Students then apply their understanding of translations, reflections, and scaling to graphs and equations of many function types. The unit ends with students applying transformations to different functions to model a real-world data set.

Section A: Translations, Reflections, and Symmetry

Section Goals

- Create equations to represent known translations and reflections of a graph.
- Justify why a function is even, odd, or neither from an equation.
- Use function notation to represent a transformation from one graph to another.

Section Narrative

In this section, students consider transformations of functions that retain the shape of the original function. They begin with vertical and horizontal translations and reflections across the horizontal and vertical axes. Students have seen these types of transformations when studying quadratic functions. This section extends that understanding to all function types. First, students consider the graphs of two possible functions as fits for a data set describing the temperature of a water bottle left outside and make an argument about why one is a better fit. Students return to this data set in future lessons as they learn more ways to transform a given equation to fit data. Next, students investigate how reflections across the horizontal and vertical axes are defined using function notation. These ideas are expanded to consider, from both a graphical and algebraic perspective, the properties of even functions, odd functions, and functions that are neither even nor odd. In parallel with their study of the effect of translations on graphs and tables, students learn to write equations for functions that are defined in terms of another function to describe transformations using function notation.

Lessons

1. **Matching Up to Data** — Let’s describe how to transform graphs.
2. **Moving Functions** — Let’s represent vertical and horizontal translations using function notation.

3. **More Movement** — Let’s translate graphs vertically and horizontally to match situations.
4. **Reflecting Functions** — Let’s reflect some graphs.
5. **Some Functions Have Symmetry** — Let's look at symmetry in graphs of functions.
6. **Symmetry in Equations** — Let’s use equations to decide if a function is even, odd, or neither.
7. **Expressing Transformations of Functions Algebraically** — Let’s express transformed functions algebraically.

Section B: Scaling Outputs and Inputs

Section Goals

- Calculate the scale factor needed to transform the input or output of a function to fit data.
- Create new functions by adding, subtracting, multiplying, and dividing functions.

Section Narrative

In this section, students explore the effects of transformations on a function that change its shape, including multiplying the input or output of a function by a scale factor and combining functions. First, they fit quadratic functions to parabolic arches in photos in order to better understand how to “squash” or “stretch” outputs as they explore the effect of multiplying the output by a scale factor. Next, students consider the change in height over time of a rider on different Ferris wheels as another application of scale factors as they explore multiplying inputs of functions by a scale factor. The use of clear and precise language is emphasized as students make sense of the effects of different scale factors. Finally, students combine functions using addition, subtraction, multiplication, and division to create a new function.

Lessons

1. **Scaling the Outputs** — Let’s stretch and squash some graphs.
2. **Scaling the Inputs** — Let’s use scale factors in different ways.
3. **Combining Functions** — Let’s make some new functions using other functions.

Section C: Transformations of Functions

Section Goals

- Create a graph of a function given an original function and a transformation.
- Create an equation for a function that has been transformed from a specific function and an unknown function f.

Section Narrative

In this section, students apply all of the work they have done with transformations so far to a variety of function types. Students transform from an original function and write equations for transformed functions, examine how transformation effects appear in equations of functions, transform parabolas, and have an optional opportunity to transform circles.

Lessons

1. **Transforming from an Original Function** — Let's transform some functions.
2. **Transformation Effects** — Let's transform some equations of functions.
3. **Transforming Parabolas** — Let's look at transformations to parabolas.
4. **Transforming Circles (Optional)** — Let's transform some circles.

Section D: Let’s Put It to Work

Section Narrative

In this final section, students have the opportunity to apply their thinking from throughout the unit. As this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions.

Lessons

1. **Making a Model for Data** — Let’s model with functions.

Unit 7: Trigonometric Functions

Unit Goal

Students build trigonometric functions by extending right-triangle ratios to the unit circle, define cosine, sine, and tangent as functions on the real numbers, and apply transformations of periodic functions to model circular and periodic situations.

Unit Narrative

In this unit, students are introduced to trigonometric functions. This unit builds directly on the work of a previous unit by having students apply their knowledge of transformations to trigonometric functions and use these functions to model periodic situations. The unit begins with a deep study of the unit circle, as students study circular motion within familiar contexts. Students then use the Pythagorean Theorem and right-triangle trigonometry to determine the coordinates of points on a circle. They use the similarity of circles and right triangles to generalize to the unit circle, focusing on the important fact that when the hypotenuse has unit length, the length of the legs can be expressed with cosine and sine. Then students transition to thinking about cosine and sine as functions. They use the unit circle to graph cosine and sine, then expand the domain to all real numbers as they learn the meaning of radian angles greater than 2π and less than 0. Students also reason about the input values where the tangent function does not exist and how the output values repeat at regular intervals, leading to the tangent function's periodic nature. Next, students apply their previous work with transformations of graphs to trigonometric functions as they identify important features of periodic functions—including midline, amplitude, and period. They apply the work of this unit by modeling periodic or approximately periodic situations both algebraically and graphically. Students create their own unit circle display in the unit. This display is meant to be a reference tool for students throughout the unit as they transition from a right-triangle-focused understanding of trigonometry to seeing cosine, sine, and tangent as functions with their own inputs and outputs.

Section A: The Unit Circle

Section Goals

- Use sine and cosine to find coordinates of points on a circle, in all four quadrants.
- Use the Pythagorean Identity to determine the value of all three trigonometric ratios, given the value of one to start from and the quadrant of the angle.

Section Narrative

This section introduces students to the unit circle as it builds on work with similar triangles and trigonometric ratios. First, students consider a context with repeating outputs. Throughout the section, students use contexts involving circular motion as they make sense of periodic functions and the unit circle. Next, students use the Pythagorean Theorem to connect legs of a right triangle to the coordinates of points on a circle. Then, students explore the unit circle over two lessons. Students build on the previous course work that led to defining the radian measure of an angle as the ratio of the arc length traveled to the radius of the circle. Students continue this work by creating a unit circle reference as they use the symmetry of the unit circle to identify radian angle measures and coordinates of points on the unit circle. Next, students connect the right triangle trigonometric ratios to the points on the circle, leading to the development of trigonometric identities over two lessons. Finally, students represent points on various circles in context, using expressions involving trigonometric ratios. This is an important transition step as students progress toward thinking about cosine and sine as functions.

Lessons

1. **Moving in Circles** — Let’s think about moving in circles.
2. **Revisiting Right Triangles** — Let’s recall and use some things we know about right triangles.
3. **The Unit Circle (Part 1)** — Let’s learn about the unit circle.
4. **The Unit Circle (Part 2)** — Let’s look at angles and points on the unit circle.
5. **The Pythagorean Identity (Part 1)** — Let’s learn more about cosine and sine.
6. **The Pythagorean Identity (Part 2)** — Let’s use the Pythagorean Identity.
7. **Finding Unknown Coordinates on a Circle** — Let’s find coordinates on a circle.

Section B: Periodic Functions

Section Goals

- Compare and contrast the features of the cosine, sine, and tangent functions.
- Interpret graphs of cosine and sine for input values between 0 and 2π, values greater than 2π, and values less than 0.

Section Narrative

This section develops sine, cosine, and tangent as trigonometric functions. This builds on work from an earlier course in which students worked with sine, cosine, and tangent ratios using right triangles, and then the work in a previous section in which students used expressions involving sine and cosine to locate points on a circle. First, students examine graphs of functions that involve repeated behavior, leading to a definition for periodic functions. Next, students begin to plot points for the sine and cosine functions, beginning with inputs from 0 to 2π radians. Students then extend this work to values greater than 2π radians, both in context and abstractly. Then, students consider inputs of negative radian values. They make sense of these negative inputs contextually, then create graphs of cosine and sine across their now extended domains. Finally, students make sense of the tangent function, using their understanding of trigonometric ratios, the unit circle, and their work with the sine and cosine functions. The last lesson in this section is an optional exploration of the secant, cosecant, and cotangent trigonometric functions.

Lessons

1. **Rising and Falling** — Let’s study graphs that repeat.
2. **Introduction to Trigonometric Functions** — Let’s graph cosine and sine.
3. **Beyond 2π** — Let’s extend our understanding of trigonometric functions.
4. **Extending the Domain of Trigonometric Functions** — Let’s think about the value of cosine and sine for all types of inputs.
5. **Tangent** — Let’s learn more about tangent.
6. **Some New Ratios (Optional)** — Let's explore graphs of some more trigonometric functions.

Section C: Trigonometry Transformations

Section Goals

- Create trigonometric functions to model circular motion, given a description.
- Identify the midline, amplitude, and period in trigonometric functions presented graphically and with equations.

Section Narrative

This section focuses on transformations of trigonometric functions. Students have seen the effects of translations and scale factors on graphs in an earlier unit when they studied general transformations of functions and in a previous course when they studied quadratic equations and their graphs. First, students are introduced to the amplitude of a periodic function in context, building from prior work with scale factor or vertical stretch of the graph of a function. Students also examine the new feature of the midline of a periodic function. They continue this work to combine changing amplitude and midline in the same periodic function as they relate to vertical stretch and vertical translations. Students also consider the effects of a horizontal translation on the equation and graph of the function. Next, students examine how a horizontal scale factor affects the period of the function. Over two lessons, they put all of these transformations together to identify and determine parameters and features of trigonometric functions, including midline, amplitude, and period, using graphs and equations. Students generalize these transformations as parameters of a periodic function, building on their work in earlier units and courses. They end the section with modeling circular motion using periodic functions.

Lessons

1. **Amplitude and Midline** — Let's transform the graphs of trigonometric functions.
2. **Transforming Trigonometric Functions** — Let’s make lots of changes to the graphs of trigonometric functions.
3. **Features of Trigonometric Graphs (Part 1)** — Let’s compare graphs and equations of trigonometric functions.
4. **Features of Trigonometric Graphs (Part 2)** — Let's explore a trigonometric function modeling a situation.
5. **Comparing Transformations** — Let's ask questions to figure out transformations of trigonometric functions.
6. **Modeling Circular Motion** — Let's use trigonometric functions to model circular motion.

Section D: Let’s Put It to Work

Section Narrative

In this final section, students have the opportunity to apply their thinking from throughout the unit. As this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions.

Lessons

1. **Beyond Circles** — Let's use trigonometric functions to model data.

Unit 8: Statistical Inferences

Unit Goal

Students examine study types and the role of randomization, model bell-shaped distributions with normal curves, quantify sampling variability with margins of error, and analyze experimental data to assess whether observed effects are statistically significant.

Unit Narrative

This unit focuses on some important uses of randomization in statistics. Initially, students consider three types of study (experimental, observational, and survey) as ways of collecting data. In each form of study, random selection of participants and assignment into any subgroups is important for being able to generalize findings to a larger population. Students notice that an estimation of a proportion or mean of a population from sample data comes with some variability because different samples can result in different estimates. This can be expressed by including a margin of error along with any point estimates given. Next, students examine the normal distribution as a common model for bell-shaped distributions. With technology, students can use a normal distribution model to approximate the proportion of data within certain intervals. This provides a way to quantify the confidence students should have in their estimates of population proportions or means. It also provides a way to check whether data from an experimental study has enough evidence to conclude that a difference in means between a control and treatment group is due to the treatment. The unit concludes with an experimental study that students can do together, from study design to data collection and analysis. Then students draw a conclusion about the experiment based on their analysis.

Section A: Study Types

Section Goals

- Understand the purpose of randomness in experimental studies, observational studies, and surveys.

Section Narrative

In this section, students examine three study types: experimental, observational, and survey. Students classify descriptions of studies as one of the three types and notice some aspects of study design that make them better at addressing a question of interest. One of the main aspects of good study design is the inclusion of randomness in the selection of participants in the study.

Lessons

1. **Being Skeptical** — Let’s examine some ways people use statistics.
2. **Study Types** — Let’s examine different kinds of studies.
3. **Randomness in Groups** — Let’s explore why randomness is important in studies.

Section B: Distributions

Section Goals

- Use a normal distribution model to estimate proportions of data within an interval.

Section Narrative

In this section, students focus on the normal distribution as a model for bell-shaped distributions. By using this model and technology (or tables of values), students can approximate the proportion of data from a distribution that is within certain intervals. This technique will be useful in later sections when students quantify expected confidence in estimates for population characteristics. The first lesson of this section is an optional review of some statistical measures, such as mean and standard deviation, and of distribution descriptions preparing students to recognize distributions that would fit with a normal distribution model.

Lessons

1. **Describing Distributions (Optional)** — Let's remember what standard deviation means and discuss distributions.
2. **Normal Distributions** — Let's investigate a specific type of distribution called a "normal distribution."
3. **Areas in Histograms** — Let's find proportions of data in certain intervals.
4. **Areas under a Normal Curve** — Let's use normal distributions to estimate proportions of data.

Section C: Not All Samples Are the Same

Section Goals

- Use data from samples to determine point estimates of population characteristics and to estimate associated margins of error.

Section Narrative

In this section, students see that samples can be used to estimate characteristics for a population and how to adjust for the fact that different samples would result in different estimates. First, students recognize that working with real data often means dealing with some amount of variability. For example, if a random sample suggests that half of a population has a trait, it might not be surprising if it is actually closer to 52% that have the trait. It might be surprising if 80% of the population does, though. Students are introduced to the idea of a margin of error that can be attached to point estimates to indicate a range of values that would not be surprising based on the information from a sample. Then students practice estimating either a proportion or mean for a population using a sample, and including a margin of error for the point estimate based on variability from simulated samples.

Lessons

1. **Not Always Ideal** — Let's see how closely data matches expectations.
2. **Variability in Samples** — Let's explore how samples can be different.
3. **Estimating Proportions from Samples** — Let's estimate population proportions with some data.
4. **Estimating a Population Mean** — Let's estimate population means, using sample data.

Section D: Analyzing Experimental Data

Section Goals

- Use the results from redistributing data from an experiment into groups to determine whether the original difference in means is significant.

Section Narrative

In this section, students examine the difference in means between two groups in an experimental study and determine if there is enough evidence to determine if the difference is likely due to the treatment. Given data from two groups in an experiment, students create a randomization distribution and use what they know about normal distributions to recognize when the observed difference in means would be unusual if there was no effect of the treatment, and thus, when the treatment that defines the group likely causes some effect on the response variable. Finally, students are given the chance to design their own experimental study, collect data, and analyze it to draw a conclusion about the experiment. A note on the language of this section: The analysis happens under the assumption that the treatment of the experiment has no effect. This means that any conclusion that can be made from the analysis either supports that assumption or does not.

Lessons

1. **Experimenting** — Let’s do an experiment.
2. **Using Normal Distributions for Experiment Analysis** — Let's determine when the results from an experiment are significant.
3. **Questioning Experimenting** — Let's ask the right questions to analyze data from an experiment.

Section E: Let's Put It to Work

Section Narrative

In this final section, students have the opportunity to apply their thinking from throughout the unit. As this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions.

Lessons

1. **Heart Rates** — Let’s collect and analyze data.

About This Syllabus

This syllabus was compiled from the publicly available Illustrative Mathematics 9-12 AGA v.360 curriculum at accessim.org. Unit and section narratives, goals, and lesson titles are sourced from the teacher view. Optional lessons are labeled “(Optional).”
Assessments (checkpoints and end-of-unit assessments) are referenced in the source curriculum but require a Kendall Hunt educator account to view; they are not included here.
https://accessim.org/9-12-aga/algebra-2?a=teacher