Algebra 1

Full Syllabus

Illustrative Mathematics® 9-12 AGA (v.360)

8 Units · Unit Goals · Section Goals · Lesson-by-Lesson Outline

Source: https://accessim.org/9-12-aga/algebra-1?a=teacher

Course Overview

Algebra 1 of the Illustrative Mathematics 9-12 AGA curriculum is organized into 8 units. Students begin with one-variable statistics, then study linear equations and systems by modeling relationships in situations. They examine relationships between variables using two-variable statistics, then solve and graph linear inequalities and systems of linear inequalities. Students deepen their understanding of functions through function notation, domain and range, average rate of change, and features of graphs, encountering linear functions and their inverses, piecewise and absolute value functions, exponential functions, and quadratic functions. The course closes with a unit on quadratic equations, where students reason about solutions, write equivalent equations, complete the square, and apply the quadratic formula.

The unit sequence is:

- Unit 1: One-Variable Statistics
- Unit 2: Linear Equations and Systems
- Unit 3: Two-Variable Statistics
- Unit 4: Linear Inequalities and Systems
- Unit 5: Functions
- Unit 6: Introduction to Exponential Functions
- Unit 7: Introduction to Quadratic Functions
- Unit 8: Quadratic Equations

Each unit is divided into lettered sections (A, B, C, D, E, F as applicable), and each section is a sequence of daily lessons. Optional lessons are noted.

Unit 1: One-Variable Statistics

Unit Goal

Students collect, display, and analyze data using statistics, such as mean, median, interquartile range, and standard deviation.

Unit Narrative

In this unit, students collect, display, and analyze data using statistics, such as mean, median, interquartile range, and standard deviation. In grades 6–8, students used histograms, dot plots, and box plots as a way to summarize data and worked with basic measures of center (mean and median) as well as measures of variability (mean absolute deviation and interquartile range). These concepts are revisited in the first two sections of this unit, but with a focus on interpretation and what they reveal about the data in addition to the mechanics of constructing the data displays. The optional third section is available to familiarize students with spreadsheets and technology that will be used to calculate statistics, such as mean, median, quartiles, and standard deviation, as well as create data displays. The fourth section introduces additional ways to interpret data using standard deviation and outliers. Students finish the unit by using these tools to compare related data sets using measures of center and measures of variability. The last lesson gives students a chance to practice their skills by posing a statistical question, designing an experiment, collecting data, and analyzing their data. Because the first half of the unit mostly revisits material from middle school, a Mid-Unit Assessment is not included in this unit. The Cool-downs and Checkpoints can be used to monitor student understanding. In this unit, only the population standard deviation is used. Sample standard deviation is introduced in a later course.

Section A: Getting to Know You

Section Goals

- Create and interpret data displays such as dot plots, histograms, and box plots.

Section Narrative

In this first section of Algebra 1, students begin to build community by collecting data from their classmates, classifying the different kinds of data they might collect, and creating displays of the information that they collected. The optional second lesson gives students a chance to practice creating dot plots, histograms, and box plots using given data before attempting to do the same with the data that they collected. This lesson would be an opportunity to introduce any technological tools the class will use for creating these displays. This section intentionally allows extra time for students to learn new routines and establish norms for the course.

Lessons

1. **Getting to Know You** — Let's work together to collect data and explore statistical questions.
2. **Data Representations (Optional)** — Let’s represent and analyze data using dot plots, histograms, and box plots.
3. **A Gallery of Data** — Let’s make, compare, and interpret data displays.

Section B: Distribution Shapes

Section Goals

- Describe the shape of a distribution, including a measure of center and a measure of variability.

Section Narrative

Students work to describe distributions of data using terms such as “symmetric,” “skewed,” “uniform,” “bimodal,” and “bell-shaped.” An optional lesson is included to remind students of prior learning about measures of center and measures of variation so that those terms can be included in distribution descriptions. Even if the optional lesson is skipped, consider creating a display about the measures of center and variability, like the one mentioned in a Synthesis of this lesson.

Lessons

1. **The Shape of Distributions** — Let’s explore data and describe distributions.
2. **Calculating Measures of Center and Variability (Optional)** — Let’s calculate measures of center and measures of variability and know which are most appropriate for the data.

Section C: How to Use Spreadsheets

Section Goals

- Use a spreadsheet as a tool for calculating values quickly.

Section Narrative

In this optional section students are introduced to using spreadsheets to create basic formulas and interpret results. It is assumed that students know nothing about spreadsheets. If students are proficient at using spreadsheets as computation tools, this sequence of lessons can be safely skipped. This technology is useful for interpreting data in this unit and is also used to examine functions in a subsequent unit. Because this entire section is optional, no checkpoint questions are included.

Lessons

1. **Mystery Computations (Optional)** — Let's explore spreadsheets.
2. **Spreadsheet Computations (Optional)** — Let's use spreadsheets as calculators.
3. **Spreadsheet Shortcuts (Optional)** — Let's explore recursive formulas in spreadsheets.

Section D: Manipulating Data

Section Goals

- Describe the effect of outliers on a distribution of data.
- Recognize standard deviation as a measure of variability.

Section Narrative

In this section, students are introduced to additional tools to analyze data such as standard deviation and outliers. They finish the section by using all of these tools to compare similar data sets. Note that in this unit, all standard deviations refer to the population standard deviation calculation rather than the sample standard deviation. If students are using GeoGebra or the Spreadsheet tool in Math Tools to create a histogram, there is an issue when the maximum value is on the boundary of the greatest interval. In this case, GeoGebra includes the maximum value with the previous interval rather than following the convention of creating a new bar for the next interval. Address this issue with students if they are using either of those tools.

Lessons

1. **Technological Graphing** — Let’s use technology to represent data.
2. **The Effect of Extremes** — Let’s see how statistics change with the data.

3. **Comparing and Contrasting Data Distributions** — Let's investigate variability using data displays and summary statistics.
4. **Standard Deviation** — Let's learn about standard deviation, another measure of variability.
5. **More Standard Deviation** — Let's continue to interpret standard deviation.
6. **Outliers** — Let's investigate outliers and how to deal with them.
7. **Comparing Data Sets** — Let's compare statistics for data sets.

Section E: Let's Put It to Work

Section Narrative

The final section of this unit is a lesson where students have the opportunity to apply their thinking from throughout the unit. As this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions.

Lessons

1. **Analyzing Data** — Let's answer statistical questions by analyzing data, and comparing and contrasting their distribution shape and measures of center and variability.

Unit 2: Linear Equations and Systems

Unit Goal

Students examine solving and graphing linear equations and systems of linear equations.

Unit Narrative

In this unit, students examine solving and graphing linear equations and systems of linear equations. The unit builds on learning from middle school when students used variables to write equations, manipulated equations using valid moves such as the distributive property, and solved basic systems of linear equations using graphs and substitution. In the first section, students recall writing equations to represent situations. In the second section, they use valid moves to write equivalent equations that can be used to solve for unknown values or to isolate variables. The third section examines solving systems of equations using graphs, substitution for variables, and elimination of variables. Students use their understanding of writing equivalent equations to understand why each of the methods works for finding the solution.

Section A: Writing and Modeling with Equations

Section Goals

- Interpret solutions to equations in one variable and in two variables.
- Recognize that the graph of a linear equation in two variables represents all pairs of values that are solutions to the equation.
- Write equations with numbers and variables to describe relationships and constraints.

Section Narrative

In this section, students recall how to write expressions and equations from situations that can vary or have unknown quantities. They begin by modeling situations using self-defined variables and constraints that fit the situation. Then, they use tables to examine patterns of values and use those to write equations. They recall what it means for a value to be a solution to an equation and then graph the equations to look at several solutions. Use the Check Your Readiness to assess students' incoming understanding of manipulating expressions and equations. If there is a solid understanding, feel free to move quickly through the optional lessons or skip them to save time.

Lessons

1. **Planning a Party** — Let's write expressions to estimate the cost of a party.
2. **Writing Equations to Model Relationships (Part 1) (Optional)** — Let's look at how equations can help us describe relationships and constraints.
3. **Writing Equations to Model Relationships (Part 2)** — Let's use patterns to help us write equations.
4. **Equations and Their Solutions (Optional)** — Let's recall what we know about solutions to equations.
5. **Equations and Their Graphs (Optional)** — Let's graph equations in two variables.

Section B: Manipulating Equations and Understanding Their Structure

Section Goals

- Determine the slope and vertical intercept of the graphs of linear equations by making use of structure or by rearranging the equations.
- Rearrange multi-variable equations to highlight a particular quantity.
- Recognize that “equivalent equations” are equations that have exactly the same solutions, and that multiple equivalent equations can represent the same relationship.

Section Narrative

In this section, students make use of equivalent equations to solve for variables. Initially, students solve equations for an unknown quantity, then later solve for a variable in terms of another variable to reinterpret situations and find values more efficiently. Then, students draw closer connections between graphs and equations in both standard and slope-intercept form.

Lessons

1. **Equivalent Equations** — Let's investigate what makes two equations equivalent.
2. **Explaining Steps for Rewriting Equations** — Let’s think about why some steps for rewriting equations are valid but other steps are not.
3. **Which Variable to Solve for? (Part 1)** — Let’s rearrange equations to pin down a certain quantity.
4. **Which Variable to Solve for? (Part 2)** — Let’s solve an equation for one of the variables.
5. **Connecting Equations to Graphs (Part 1)** — Let’s investigate what graphs can tell us about the equations and relationships they represent.
6. **Connecting Equations to Graphs (Part 2)** — Let's analyze different forms of linear equations and how the forms relate to their graphs.

Section C: Systems of Linear Equations in Two Variables

Section Goals

- Determine whether a system of equations will have 0, 1, or infinitely many solutions by analyzing their structure or by graphing.
- Use elimination or substitution to create one or more equivalent systems to help solve the original system.

Section Narrative

In this section, students revisit systems of equations and develop algebraic methods for finding any solutions. In earlier grades, students explored solving systems using graphs and substitution methods. In addition to revisiting these concepts, students use what they have learned in previous sections about writing equivalent equations to solve systems by using elimination methods. After looking at solutions to systems on a graph, students use substitution and elimination methods to solve the systems. They see that equivalent equations have different graphs, but continue to intersect the other graphs at the solution to the system. The final lesson of the section looks, graphically as well as algebraically, at linear systems that have no solution or infinitely many solutions.

Lessons

1. **Writing and Graphing Systems of Linear Equations** — Let’s recall what it means to solve a system of linear equations and how to do it by graphing.
2. **Solving Systems by Substitution** — Let’s use substitution to solve systems of linear equations.

3. **Solving Systems by Elimination (Part 1)** — Let’s investigate how adding or subtracting equations can help us solve systems of linear equations.
4. **Solving Systems by Elimination (Part 2)** — Let’s think about why adding and subtracting equations work for solving systems of linear equations.
5. **Solving Systems by Elimination (Part 3)** — Let's find out how multiplying equations by a factor can help us solve systems of linear equations.
6. **Systems of Linear Equations and Their Solutions** — Let's find out how many solutions a system of equations could have.

Section D: Let's Put It to Work

Section Narrative

In the final section, students have the opportunity to apply their thinking from throughout the unit. Because this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions.

Lessons

1. **Asking about Solving Systems (Optional)** — Let's figure out some systems.
2. **Linear Patterns (Optional)** — Let’s graph and explore systems that make a pattern.

Unit 3: Two-Variable Statistics

Unit Goal

Students use statistical methods to look for associations in bivariate data, working with two-way tables, scatter plots, lines of best fit, residuals, and correlation coefficients.

Unit Narrative

In this unit, students use statistical methods to look for associations in bivariate data. The unit begins with students analyzing categorical data arranged in two-way tables. Students use the relative frequencies of the combinations of those categorical variables to check for evidence of any associations in the data. The unit then transitions to bivariate numerical data, which are visualized using scatter plots and lines of best fit. Students use technology to compute the lines of best fit and observe how well the linear models match the data. Residuals and correlation coefficients are used to quantify the goodness of fit for linear models. The unit closes with an exploration of the difference between correlation and causal relationships, and it is also an opportunity to apply this learning to areas of interest, like anthropology and sports. In grade 8, students informally constructed scatter plots and lines of fit, noticed linear patterns, and observed associations in categorical data using two-way tables. In this unit, students build on this previous knowledge by assessing how well a linear model matches the data by using residuals as well as the correlation coefficient for best-fit lines (found using technology). There are opportunities to practice concepts from a previous unit by interpreting the slope and intercept of a linear model in context as well as using the models to predict one variable given information about the other.

Section A: Two-Way Tables

Section Goals

- Create relative frequency tables from information given in a two-way table or about a situation.
- Inspect patterns in relative frequency tables and two-way tables to determine if there is a possible association between two variables of interest.

Section Narrative

This section focuses on interpreting categorical data using two-way tables. First, students recall from grade 8 how to interpret the values in a two-way table. Then, they expand on their understanding by creating relative frequency tables and use those to look for an association between the variables.

Lessons

1. **Two-Way Tables** — Let's look at categorical data.
2. **Relative Frequency Tables** — Let's find relative frequencies of categorical data.
3. **Associations in Categorical Data** — Let's look for relationships between categorical variables.

Section B: Scatter Plots

Section Goals

- Comprehend the connection between residuals, variability, and whether or not using a linear model is appropriate.
- Interpret the rate of change and vertical intercept for a linear model in the context of a situation.

Section Narrative

In this section, students focus on linear models and how well they fit data. The first lesson motivates the importance of having a linear model by making sense of the slope and intercept in context and showing how to predict additional values from the model. The next two lessons are about informally assessing how to obtain the linear models. Students progress from selecting between given models to estimating their own models to using technology in order to find a best-fit line. They then use residuals to more quantitatively understand how well a linear model fits data.

Lessons

1. **Linear Models** — Let's explore relationships between two numerical variables.
2. **Fitting Lines** — Let's find the best linear model for some data.
3. **Residuals** — Let's examine how close data is to linear models.

Section C: Correlation Coefficients

Section Goals

- Describe the strength and sign of the relationship between variables based on the correlation coefficient.
- Investigate the relationship between two variables to analyze whether or not the relationship is causal.

Section Narrative

In this section, students look more deeply at the relationships between two variables. In particular, they use the correlation coefficient to categorize the relationships as strong or weak and as positive or negative. They then use their understanding of situations to suggest whether the relationship is causal or merely a correlation.

Lessons

1. **The Correlation Coefficient** — Let's see how good a linear model is for some data.
2. **Using the Correlation Coefficient** — Let's look closer at correlation coefficients.
3. **Causal Relationships** — Let's get a closer look at related variables.

Section D: Let's Put It to Work

Section Narrative

In the final section, students have the opportunity to apply their thinking from throughout the unit. As this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions.

Lessons

1. **Fossils and Flags (Optional)** — Let's collect some data and analyze it.

Unit 4: Linear Inequalities and Systems

Unit Goal

Students examine solving and graphing linear inequalities and systems of linear inequalities.

Unit Narrative

In this unit, students examine solving and graphing linear inequalities and systems of linear inequalities. The unit builds on concepts from middle school when students write and solve inequalities by reasoning about quantities. It further builds on concepts from an earlier unit in which students solve linear equations and systems of equations by writing equivalent equations. To start, students solve linear inequalities in one variable and graph the solutions on a number line by writing equivalent inequalities. In the second section, they solve linear inequalities with two variables by looking at the related equation, graphing it on a coordinate plane, and testing points on either side of the line to determine the solution region. The third section is about solving systems of linear inequalities considering multiple linear inequalities as conditions for situations and finding a solution region that satisfies all of the inequalities.

Section A: Linear Inequalities in One Variable

Section Goals

- Use a related equation to solve an inequality in one variable.
- Write and solve inequalities in one variable to represent the constraints in situations and to solve problems.

Section Narrative

In this section, students solve linear inequalities in one variable. They begin by writing constraints for situations as inequalities, and then solve them by writing equivalent inequalities and reasoning about quantities. They finish the section by examining inequalities for constraints that are implied by the context, such as requiring variables to be positive. Much of the work of this section revisits concepts from middle school, but the reasoning in this unit builds on the work that students did with equations in an earlier unit. Depending on student proficiency in solving inequalities with one variable, this section can either be moved through quickly or extra time can be spent on several optional activities available for additional practice solving inequalities.

Lessons

1. **Representing Situations with Inequalities** — Let's use inequalities to represent constraints in situations.
2. **Solutions to Inequalities in One Variable** — Let's find and interpret solutions to inequalities in one variable.
3. **Writing and Solving Inequalities in One Variable** — Let's solve problems by writing and solving inequalities in one variable.

Section B: Linear Inequalities in Two Variables

Section Goals

- Understand that a constraint on two variables can be represented by an inequality, a graph (a half-plane), and a verbal description.

- Write inequalities in two variables to represent the constraints in a situation, and use technology to graph the solution set to answer questions about the situation.

Section Narrative

In this section, students work with inequalities that have two variables. They begin by considering what it means to be a solution of an inequality of this type, and then recognize that, on a graph, the solutions occupy a region bounded by a linear equation related to the original inequality. This understanding is used to graph solution regions for 2-variable inequalities—first by graphing an equation related to the inequality, and then by testing points to find the solution region. Initially, students work through this process by hand, but later they are introduced to the use of graphing technology to speed up the process.

Lessons

1. **Graphing Linear Inequalities in Two Variables (Part 1)** — Let's find out how to use graphs to represent solutions to inequalities in two variables.
2. **Graphing Linear Inequalities in Two Variables (Part 2)** — Let's write inequalities in two variables and make sense of the solutions by reasoning and by graphing.
3. **Solving Problems with Inequalities in Two Variables** — Let's practice writing, interpreting, and graphing solutions to inequalities in two variables.

Section C: Systems of Linear Inequalities in Two Variables

Section Goals

- Given a system of inequalities and their graphs, explain how to tell if a pair of values is a solution to the system.
- Understand that the solution set of a system of inequalities in two variables is composed of any pair of values that make both inequalities true, and that it is represented graphically by the region where the graphs overlap.

Section Narrative

This section expands the exploration of inequalities to include systems. Students begin by interpreting solutions to systems of inequalities as values that satisfy all the constraints simultaneously. Then, they graph the inequalities to visualize a solution region for the systems as the overlapping portions of the solution regions for the individual inequalities. In the optional last lesson of the unit, students mathematically model a situation using systems of linear inequalities and explore the solutions in that context.

Lessons

1. **Solutions to Systems of Linear Inequalities in Two Variables** — Let's look at situations in which two constraints (that can be expressed by inequalities) must be met simultaneously.
2. **Solving Problems with Systems of Linear Inequalities in Two Variables** — Let's use systems of inequalities to solve some problems.
3. **Modeling with Systems of Inequalities in Two Variables** — Let's create mathematical models using systems of inequalities.

Unit 5: Functions

Unit Goal

Students expand and deepen their understanding of functions, working with function notation, features of graphs, domain and range, piecewise and absolute value functions, and inverse functions.

Unit Narrative

In this unit, students expand and deepen their understanding of functions. They begin with a reminder of the definition of a function (a rule that assigns exactly one output to each input) that they previously saw in grade 8, then get familiar with function notation and use it to compare and analyze functions, write rules for functions, and solve for inputs or outputs. Then, students explore graphs of functions to describe features such as "maximum," "minimum," "intercepts," "increasing," "decreasing," and "average rate of change" and make connections between the graphs and real-life situations. They use situations to discuss the domain and range of a function and make sense of piecewise-defined functions. In particular, students examine the absolute value function and some basic transformations of it. Later, students explore inverses of linear functions as a way to find corresponding input values when output values are known. Throughout the unit, students have chances to mathematically model real-world situations.

Section A: Functions and Their Representations

Section Goals

- Sketch a graph of a function given statements in function notation.
- Understand that a relationship between two quantities is a function if there is only one possible output for each input.
- Write equations that represent rules using function notation.

Section Narrative

In this section, students recall the definition of a function as a rule that assigns no more than one output to every input. Then, they use function notation to describe rules from real-world situations and understand how the notation differentiates between input and output values.

Lessons

1. **Describing and Graphing Situations** — Let's look at some fun functions around us and try to describe them!
2. **Function Notation** — Let's learn about a handy way to refer to and talk about a function.
3. **Interpreting and Using Function Notation** — Let's use function notation to talk about functions.
4. **Using Function Notation to Describe Rules (Part 1)** — Let's use function notation to describe rules.
5. **Using Function Notation to Describe Rules (Part 2)** — Let's graph and find the values of some functions.

Section B: Analyzing and Creating Graphs of Functions

Section Goals

- Given a graph of a function, estimate or calculate the average rate of change over a specified interval.

- Interpret key features of a graph—the intercepts, maximums, minimums, and intervals when the function is increasing or decreasing—in terms of a situation.
- Interpret statements about two or more functions written in function notation.

Section Narrative

In this section, students analyze graphs of functions including special features, such as "maximum," "minimum," "intercepts," and "average rate of change." They make connections between descriptions of real-life situations and graphs of functions that model those situations. Then they compare situations using graphs and function notation.

Lessons

1. **Features of Graphs** — Let's use graphs of functions to learn about situations.
2. **Using Graphs to Find Average Rate of Change** — Let's measure how quickly the output of a function changes.
3. **Interpreting and Creating Graphs** — Let's sketch graphs to represent situations.
4. **Comparing Graphs** — Let's compare graphs of functions to learn about the situations they represent.

Section C: A Closer Look at Inputs and Outputs

Section Goals

- Interpret an absolute value function described in words or in function notation, and create a table of values and a graph to represent the function.
- Interpret the graph of a piecewise function or its rules given in function notation, and explain the rules (orally and in writing) in terms of a situation.
- Understand that the domain of a function is the set of all possible inputs and the range is the set of all possible outputs.

Section Narrative

In this section, students focus on the domain and range of various functions. The language of domain and range allows students to better understand piecewise functions for which different rules apply to different parts of the domain. In particular, students explore absolute value functions by way of a situation in which students determine how far away guesses are from an actual value.

Lessons

1. **Domain and Range (Part 1)** — Let's find all possible inputs and outputs for a function.
2. **Domain and Range (Part 2)** — Let's analyze graphs of functions to learn about their domain and range.
3. **Piecewise Functions** — Let's look at functions that are defined in pieces.
4. **Absolute Value Functions (Part 1)** — Let's make some guesses and see how good they are.
5. **Absolute Value Functions (Part 2)** — Let's investigate distance as a function.

Section D: Inverse Functions

Section Goals

- Find the inverse of a linear function by solving an equation for the input variable.
- Write a linear function and an inverse function to model data and solve problems.

Section Narrative

In this section, students work with inverse functions as a way to find input values that correspond to known output values. Initially, students use the idea of writing a secret code to motivate the need to find the input that produced a given output. Then, they are provided contexts in which it is efficient to use the inverse to find inputs when several different outputs are known, such as converting between currencies or temperature scales.

Lessons

1. **Inverse Functions** — Let's define functions forward and backward.
2. **Finding and Interpreting Inverse Functions** — Let's find the inverses of linear functions.
3. **Writing Inverse Functions to Solve Problems** — Let's use inverse functions to solve problems.

Section E: Let's Put It to Work

Section Narrative

In this final section, students have the opportunity to apply their thinking from throughout the unit. As this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions.

Lessons

1. **Using Functions to Model Battery Power** — Let's use functions to model data and make predictions.

Unit 6: Introduction to Exponential Functions

Unit Goal

Students build on their understanding of linear functions, properties of exponents, and percent change to explore exponential relationships, write and graph exponential functions, and compare them with linear functions.

Unit Narrative

In this unit, students build on their understanding of linear functions, properties of exponents, and percent change to explore exponential relationships. Students learn that exponential relationships are characterized by a constant quotient over equal intervals, and compare it to linear relationships, which are characterized by a constant difference over equal intervals. They encounter contexts that change exponentially. These contexts are presented verbally and with tables and graphs. They construct equations and use them to model situations and solve problems. At first, students investigate these exponential relationships without using function notation and language so that they can focus on gaining an appreciation for critical properties and characteristics of exponential relationships. Later, students view these relationships as functions and employ the notation and terminology of functions. They study graphs of exponential functions both in terms of contexts that they represent and abstract functions that don't represent a particular context, observing the effect of different parameter values on the graph. The contexts used early in this unit lead to functions where the domain is the integers. In some situations, students will interpret exponential expressions with fractional values in the exponent using graphing, but the connection to roots or logarithms is left for a later course. Note on materials: Throughout this unit, students should have access to a calculator with an exponent button. Access to graphing technology is necessary for some activities and encouraged throughout the unit.

Section A: Looking at Growth

Section Goals

- Compare linear and exponential relationships by performing calculations.
- Describe patterns in tables that represent linear and exponential relationships.

Section Narrative

In this section, students begin their exploration of exponential functions. They consider patterns of numbers that increase by a common factor and compare them to patterns that increase by a common difference. This motivates the need for a way to describe these non-linear patterns with exponential expressions.

Lessons

1. **Growing and Growing** — Let's choose the better deal.
2. **Patterns of Growth** — Let's compare different patterns of growth.

Section B: A New Kind of Relationship

Section Goals

- Interpret a negative exponent in equations that represent exponential growth or decay.
- Write an equation to represent a quantity that changes by a growth factor.
- Write and graph an equation that represents exponential decay to solve problems.

Section Narrative

In this section, students write expressions to represent exponential growth. They make use of properties of exponents and repeated multiplication of a growth factor. They are introduced to patterns that decrease in an exponential way, by looking at growth factors between 0 and 1 and by rewriting expressions using the distributive property. Later in the section, students examine graphs of exponential patterns and patterns to consider the meaning of negative exponents in context. Students must rely on properties of exponents, first developed in grade 8, to create a consistent understanding of negative exponents. Throughout the section, students continue to make connections between descriptions of situations, tables of values, graphs, and expressions or equations representing exponential change.

Lessons

1. **Representing Exponential Growth** — Let's explore exponential growth.
2. **Representing Exponential Decay** — Let's think about how to show and talk about exponential decay.
3. **Understanding Decay** — Let's look at exponential decay.
4. **Analyzing Graphs** — Let's compare situations in which quantities change exponentially.
5. **Using Negative Exponents** — Let's look more closely at exponential graphs and equations.

Section C: Exponential Functions

Section Goals

- Describe the effect of changing parameters on a graph that represents an exponential function.
- Use function notation to write equations that represent exponential relationships.

Section Narrative

In this section, students begin writing exponential relationships as functions. They revisit function notation and apply language associated with functions, such as dependent and independent variables. They also consider average rate of change in an exponential context and conclude that these measures are less representative of the function than when using linear functions. Later, students examine the impact of parameters on functions of the form and apply that in order to mathematically model situations that change exponentially. The last lesson in the section is optional and provides additional practice exploring the parameters in functions.

Lessons

1. **Exponential Situations as Functions** — Let's explore exponential functions.
2. **Interpreting Exponential Functions** — Let's find some meaningful ways to represent exponential functions.
3. **Looking at Rates of Change** — Let's calculate average rates of change for exponential functions.
4. **Modeling Exponential Behavior** — Let's use exponential functions to model real-life situations.
5. **Reasoning about Exponential Graphs (Part 1)** — Let's study and compare equations and graphs of exponential functions.
6. **Reasoning about Exponential Graphs (Part 2) (Optional)** — Let's investigate what we can learn from graphs that represent exponential functions.

Section D: Percent Growth and Decay

Section Goals

- Calculate the result of repeated percent increase for the same initial balance and interest rate, but compounded at different intervals.
- Justify why applying a percent increase n times is not equivalent to applying the percent n times.

Section Narrative

In this section, students examine repeated percent change and connect that to exponential functions. They begin by recalling percent change, then explore situations where percent changes are repeated over equal time intervals. Students then look at compound interest and how different compounding intervals affect the final value. They also write equivalent exponential expressions to compare growth or decay over different intervals.

Lessons

1. **Recalling Percent Change (Optional)** — Let's find the result of changing a number by a percentage.
2. **Functions Involving Percent Change** — Let's investigate what happens when we repeatedly apply a percent increase to a quantity.
3. **Compounding Interest** — Let's explore different ways of repeatedly applying a percent increase.
4. **Different Compounding Intervals** — Let's find out what happens when we repeatedly apply the same percent increase at different intervals of time.
5. **Expressed in Different Ways** — Let's write exponential expressions in different ways.

Section E: Comparing Linear and Exponential Functions

Section Goals

- Use rates of change, and show that, for any equal intervals of the independent variable, an exponential function always increases or decreases by an equal factor.
- Use tables, calculations, and graphs to compare growth rates of linear and exponential functions.

Section Narrative

In this section, students revisit comparisons between linear and exponential change, now with the focus on functions. They see that exponential functions eventually grow more quickly than do linear functions. They also justify that linear functions have a constant difference between any 2 inputs that are 1 unit apart, while exponential functions have a constant ratio between any 2 inputs that are 1 unit apart.

Lessons

1. **Which One Changes Faster?** — Let's compare linear and exponential functions as they continue to increase.
2. **Changes over Equal Intervals** — Let's explore how linear and exponential functions change over equal intervals.

Section F: Let's Put It to Work

Section Narrative

In this final section, students have the opportunity to apply their thinking from throughout the unit. As this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions.

Lessons

1. **Predicting Populations** — Let's use linear and exponential models to represent and understand population changes.

Unit 7: Introduction to Quadratic Functions

Unit Goal

Students look at patterns that grow quadratically, contrast quadratic growth with linear and exponential growth, and examine standard, factored, and vertex forms of quadratic functions.

Unit Narrative

Prior to this unit, students have studied what it means for a relationship to be a function, used function notation, and investigated linear and exponential functions. In this unit, they look at some patterns that grow quadratically and contrast this growth with linear and exponential growth. They further observe that eventually these quadratic patterns grow more quickly than do linear patterns but more slowly than exponential patterns grow. Students examine the important example of free-falling objects whose height over time can be modeled with quadratic functions. They use tables, graphs, and equations to describe the movement of these objects, eventually looking at the situation in which a projectile is launched upward. They interpret the meaning of each term in this context and work toward understanding how the coefficients influence the shape of the graph. Additional situations, such as revenue and area, are also introduced. Next, students examine standard, factored, and vertex forms of quadratic functions. They recognize what information about the graph is easily obtained from each form and how the different values in each form influence the graph. In particular, they begin to generalize ideas of how horizontal and vertical translation, as well as vertical and horizontal stretching of graphs, relate to modifying the equation of a function.

Section A: A Different Kind of Change

Section Goals

- Comprehend that a “quadratic relationship” can be expressed with a squared term.
- Determine and explain whether a visual pattern represents a linear, exponential, or quadratic relationship.

Section Narrative

In this section, students are introduced to patterns that grow quadratically. They compare these new patterns to linear and exponential patterns from earlier units and recognize that a new type of relationship is needed. They work with rectangular areas that have a given perimeter and visual patterns that grow quadratically with each step and use tables, graphs, and expressions to describe the relationships. Throughout the section there are opportunities to use technology to solve problems. If possible, make these tools available so that students can focus on the pattern itself rather than calculations or visualization.

Lessons

1. **A Different Kind of Change** — Let’s find the rectangle with the greatest area.
2. **How Does It Change?** — Let’s describe some patterns of change.

Section B: Quadratic Functions

Section Goals

- Interpret quadratic functions that represent a physical phenomenon, given expressions and graphs.

- Use graphs, tables, and calculations to show that exponential functions eventually overtake quadratic functions.

Section Narrative

In this section, students examine quadratic relationships in the context of functions. They compare quadratic-function growth to that of linear and exponential functions. They notice that quadratic functions with a positive quadratic coefficient grow faster than linear functions and grow slower than exponential functions that have a growth factor greater than 1. Then, students explore quadratic functions in contexts such as objects in free-fall, projectile motion, and revenue curves. They use the structure of the functions to determine the zeros, intercepts, and vertex.

Lessons

1. **Building Quadratic Functions from Geometric Patterns** — Let's describe some other geometric patterns.
2. **Comparing Quadratic and Exponential Functions** — Let's compare quadratic and exponential changes and see which one grows faster.
3. **Building Quadratic Functions to Describe Situations (Part 1)** — Let's measure falling objects.
4. **Building Quadratic Functions to Describe Situations (Part 2)** — Let's look at the objects being launched in the air.
5. **Building Quadratic Functions to Describe Situations (Part 3)** — Let's look at how to maximize revenue.

Section C: Working with Quadratic Expressions

Section Goals

- Coordinate a quadratic expression given in factored form and the intercepts of its graph.
- Use the distributive property to write equivalent quadratic expressions from factored into standard form.

Section Narrative

In this section, students formally recognize that there are multiple useful forms for quadratic expressions and functions. They examine standard and factored forms and use the distributive property to recognize the same relationship in both forms. At first, they use an area model, familiar from earlier grades, and similar diagrams to do the distribution, but are encouraged later to work without the diagram. Then they connect the structure of different forms of the equations to features of the graph, including the intercepts and vertex.

Lessons

1. **Equivalent Quadratic Expressions** — Let's use diagrams to help us rewrite quadratic expressions.
2. **Standard Form and Factored Form** — Let's write quadratic expressions in different forms.
3. **Graphs of Functions in Standard and Factored Forms** — Let's find out what quadratic expressions in standard and factored forms can reveal about the properties of their graphs.

Section D: Features of Graphs of Quadratic Functions

Section Goals

- Explain how a graph is affected by changing parameters in quadratic expressions written in standard, factored, and vertex forms.
- Use an equation in vertex form to identify the maximum or minimum of a quadratic function.

Section Narrative

In this section, students begin by exploring the effect on their graphs of the different parameters in factored and standard forms of quadratic functions. They recognize the importance of the vertex of a parabola and are introduced to the vertex form of a quadratic expression. Then they use the vertex form to examine how modifying a function can lead to translating the graph in the coordinate plane. Several optional lessons are available for students to explore deeper or to practice what they have learned. Students look at the effect of the quadratic coefficient and linear term of a quadratic function in standard form, but an optional lesson is available for students curious about the linear coefficient. There is also an optional practice lesson available for revisiting quadratic functions in context and another about using the parameters in a function to determine whether the graph will open upward or downward.

Lessons

1. **Graphing from the Factored Form** — Let's graph some quadratic functions in factored form.
2. **Graphing the Standard Form (Part 1)** — Let's see how the numbers in expressions like these affect their graph.
3. **Graphing the Standard Form (Part 2) (Optional)** — Let's change some other parts of a quadratic expression and see how they affect its graph.
4. **Graphs That Represent Situations (Optional)** — Let's examine graphs that represent the paths of objects being launched into the air.
5. **Vertex Form** — Let's find out about the vertex form.
6. **Graphing from the Vertex Form (Optional)** — Let's graph equations in vertex form.
7. **Changing the Vertex** — Let's write new quadratic equations in vertex form to produce certain graphs.

Unit 8: Quadratic Equations

Unit Goal

Students interpret, write, and solve quadratic equations algebraically using factoring, completing the square, and the quadratic formula, and use vertex form to find maximum and minimum values.

Unit Narrative

In this unit, students interpret, write, and solve equations algebraically. Previously, students represented quadratic functions using expressions, tables, and descriptions. They connected important features of graphs to standard, factored, and vertex forms and expanded expressions into standard form. Students begin with solving quadratic equations through reasoning without much algebraic manipulation. Then, they examine solving using the zero product property for quadratic equations that can be written in factored form. They notice patterns that help them rewrite quadratic expressions in factored form and recognize that not all of them are easily factorable. This motivates finding another process of solving quadratic equations. Students recognize the benefits of equations of certain forms and develop the method of completing the square. Then, they generalize this process to the quadratic formula. Throughout the unit, students analyze quadratic equations to recognize that there can be 0, 1, or 2 solutions. Solutions can be rational, irrational, or combinations of these. Students interpret the solutions that arise in different contexts. The unit concludes with rewriting quadratic expressions from standard form into vertex form to find maximum and minimum values, then apply all of their understanding to applied problems.

Section A: Finding Unknown Inputs

Section Goals

- Write quadratic equations, and reason about their solutions in terms of a situation.

Section Narrative

In this section, students are presented with activities that motivate the need for more algebraic methods for solving quadratic equations. Students write quadratic equations from situations, but recognize that they do not have the tools to accurately solve the equations unless they are in very specific forms, such as a factored form equal to zero or a perfect square equal to another perfect square.

Lessons

1. **Finding Unknown Inputs** — Let's find some new equations to solve.
2. **When and Why Do We Write Quadratic Equations?** — Let's try to solve some quadratic equations.

Section B: Solving Quadratic Equations

Section Goals

- Given a quadratic expression, create an equivalent expression in factored form.
- Use factored form and the zero product property to solve quadratic equations.

Section Narrative

In this section, students look more deeply into solving quadratic equations that can be written as a factored form equal to zero. They formalize their understanding of the zero product property, then work

on recognizing patterns that allow them to rewrite quadratic expressions from standard form to factored form. Students also begin to recognize that quadratic equations can have 0, 1, or 2 solutions. Note that this course includes only a few lessons on transforming quadratic expressions from standard form to factored form. This is intentional. The goal is to help students see factored form conceptually, understand what it can tell us about the function, and use that knowledge in modeling problems, including problems where the zeros are not rational and algebraic factoring wouldn't help. Too much attention to the algebraic skill of factoring can obscure these underlying concepts. Sometimes—including in later courses and beyond—expressions do need to be written in factored form. With the increasing availability of computer algebra systems, however, there is less need to spend lots of time learning how to factor by hand.

Lessons

1. **Solving Quadratic Equations by Reasoning** — Let's find solutions to quadratic equations.
2. **Solving Quadratic Equations with the Zero Product Property** — Let's find solutions to equations that contain products that equal zero.
3. **How Many Solutions?** — Let's use graphs to investigate quadratic equations that have two solutions, one solution, or no solutions.
4. **Rewriting Quadratic Expressions in Factored Form (Part 1)** — Let's write expressions in factored form.
5. **Rewriting Quadratic Expressions in Factored Form (Part 2)** — Let's write some more expressions in factored form.
6. **Rewriting Quadratic Expressions in Factored Form (Part 3)** — Let's look closely at some special kinds of factors.
7. **Solving Quadratic Equations by Using Factored Form** — Let's solve some quadratic equations that, before now, we could only solve by graphing.
8. **Rewriting Quadratic Expressions in Factored Form (Part 4)** — Let's transform more-complicated quadratic expressions into the factored form.

Section C: Completing the Square

Section Goals

- Comprehend that the "plus-minus" symbol is used to represent both square roots of a number and that the square root notation expresses only the positive square root.
- Solve quadratic equations by rearranging terms and completing the square.

Section Narrative

In this section, students learn to complete the square as an additional strategy for solving quadratic equations. First, they begin by recognizing when quadratic expressions can be rewritten as perfect squares and how this form is useful for solving equations. Then, students learn how to turn any quadratic equation into that form by completing the square.

Lessons

1. **What Are Perfect Squares?** — Let's see how perfect squares make some equations easier to solve.
2. **Completing the Square (Part 1)** — Let's learn a new method for solving quadratic equations.
3. **Completing the Square (Part 2)** — Let's solve some harder quadratic equations.

4. **Completing the Square (Part 3)** — Let's complete the square for some more complicated expressions.
5. **Quadratic Equations with Irrational Solutions** — Let's find exact solutions to quadratic equations, even if the solutions are irrational.

Section D: The Quadratic Formula

Section Goals

- Explain whether the products or sums of combinations of rational and irrational numbers are rational or irrational.
- Use the quadratic formula to solve quadratic equations.

Section Narrative

In this section, students encounter the quadratic formula. Initially, the formula is given to students, and they notice that the solutions match those found using other methods. After getting comfortable with using the formula, they examine a proof of the quadratic formula that uses completing the square. An optional lesson in which students analyze common errors encountered using the quadratic formula is included. If students are also making similar mistakes, consider making time to include the lesson. The section ends with a look at rational and irrational numbers first encountered in grade 8. Here, they classify sums and products of various combinations of these numbers as either irrational or rational.

Lessons

1. **The Quadratic Formula** — Let's learn a formula for finding solutions to quadratic equations.
2. **Applying the Quadratic Formula (Part 1)** — Let's use the quadratic formula to solve some problems.
3. **Applying the Quadratic Formula (Part 2) (Optional)** — Let's use the quadratic formula and solve quadratic equations with care.
4. **Deriving the Quadratic Formula** — Let's derive the quadratic formula.
5. **Rational and Irrational Solutions** — Let's consider the kinds of numbers we get when solving quadratic equations.
6. **Sums and Products of Rational and Irrational Numbers** — Let's make convincing arguments about why the sums and products of rational numbers and irrational numbers are always certain kinds of numbers.

Section E: Vertex Form Revisited

Section Goals

- Rewrite a quadratic expression in vertex form to identify the maximum or minimum value of the function that the expression defines.

Section Narrative

In this section, students revisit the vertex form of quadratic expressions. In a previous unit, students noticed that this form can be used to easily find the vertex of a quadratic function and identified that point on a graph. Here, students use the process of completing the square to rewrite expressions from standard form to vertex form, then classify the vertex as either a maximum or minimum and justify their reasoning.

Lessons

1. **Rewriting Quadratic Expressions in Vertex Form** — Let's see what else completing the square can help us do.
2. **Using Quadratic Expressions in Vertex Form to Solve Problems** — Let's find the maximum or minimum value of a quadratic function.

Section F: Let's Put It to Work

Section Narrative

The final section of this unit is a lesson where students have the opportunity to apply their thinking from throughout the unit. As this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions.

Lessons

1. **Using Quadratic Equations to Model Situations and Solve Problems** — Let's analyze a situation modeled by a quadratic equation.

About This Syllabus

This syllabus was compiled from the publicly available Illustrative Mathematics 9-12 AGA v.360 curriculum at accessim.org. Unit and section narratives, goals, and lesson titles are sourced from the teacher view. Optional lessons are labeled “(Optional).”
Assessments (checkpoints, mid-unit, and end-of-unit assessments) are referenced in the source curriculum but require a Kendall Hunt educator account to view; they are not included here.
https://accessim.org/9-12-aga/algebra-1?a=teacher