

Grade 5 • Unit 2 • Assessment Guide

Fractions as Quotients and Fraction Multiplication

For the Parent or Teacher — What to look for

Unit 2 builds two crucial fifth-grade ideas: (1) a fraction IS a division — a/b means a divided by b — and (2) multiplying a whole number by a fraction can shrink OR grow the original. Watch for whether the student can move between division, fraction, and area-of-rectangle interpretations.

Problem 1 — 5 children, 3 sandwiches

- Each child gets $3/5$ of a sandwich.
- Equation: $3 \div 5 = 3/5$.
- ★ THE foundational result of this unit. If they answer $5/3$ they have swapped numerator and denominator — they're treating it as 'more things than people.'
- Strong drawing: each of 3 sandwiches cut into 5 pieces, one piece given to each child — they get 3 pieces each.

Problem 2 — $7/8$ vs $9/8$ without computing

- $9/8$ is bigger.
- ★ Strong reasoning: 'Same number of sharers (8), but more to share (9 instead of 7), so each share is bigger.'
- Weak: actually computes (defeats the purpose), or thinks $7/8$ is bigger because 'the smaller number on top means each piece is bigger' (confused logic).

Problem 3 — Diego vs Sara on $4/3$

- Diego is correct. $4 \div 3 = 4/3 = 1$ and $1/3$.
- ★ Sara has the classic misconception that division 'turns numbers around' — actually it is the SAME order as the fraction (dividend over divisor).
- Strong: 'Diego is right. The first number (4) stays on top.'

Problem 4 — $6 \times 2/3$ two ways

- Answer: $12/3 = 4$.
- Way 1 (repeated addition): $2/3 + 2/3 + 2/3 + 2/3 + 2/3 + 2/3 = 12/3 = 4$.
- Way 2 (using division): $2/3 = 2 \div 3$, so $6 \times (2 \div 3) = 12 \div 3 = 4$. (Or: $6 \times 2 = 12$, then divided by $3 = 4$.)
- ★ Strong: shows BOTH methods and arrives at 4. Reveals the connection between multiplication and division.

Problem 5 — $5 \times 3/4$

- Answer: $15/4 = 3$ and $3/4$ miles.
- ★ Strong: writes the equation, shows the multiplication, converts to mixed number cleanly.
- Weak: stops at $15/4$ without converting; or computes $5 \times 3 = 15$ and writes 15 miles.

Problem 6 — Is $8 \times 3/5$ greater or less than 8?

- LESS than 8 (because $3/5 < 1$, multiplying by $3/5$ shrinks).
- ★ Strong reasoning: ' $3/5$ is less than 1, so we're taking $3/5$ OF 8, not all of 8 — so the result is smaller.'
- ★ This question fights the common kid-belief that 'multiplication always makes things bigger.'

- Weak: says 'greater' because 'multiplication makes things bigger,' OR computes exactly and answers 4.8 but cannot articulate why.

Problem 7 — Distributive: 4×2 and $1/2$

- Area = $(4 \times 2) + (4 \times 1/2) = 8 + 2 = 10$ sq units.
- ★ Strong: explains that you can split the rectangle into a 4×2 piece and a $4 \times (1/2)$ piece, find each area, then add.
- Weak: gets the right number but cannot explain WHY decomposing works.

Problem 8 — Pat's misconception: multiplication always shrinks

- Pat is WRONG. Multiplying by a number GREATER than 1 (like $7/3$) MAKES IT BIGGER. $5 \times 7/3 = 35/3 \approx 11.67$.
- ★ The key principle: multiplying by a fraction < 1 shrinks; multiplying by a fraction > 1 grows.
- Strong: states this rule clearly and gives examples in both directions.
- Weak: agrees with Pat, or cannot articulate when multiplication grows vs shrinks.

Problem 9 — Diagram for $8/5 = 8 / 5$

- Each child gets $8/5 = 1$ and $3/5$ candy bars.
- Strong drawing: 8 bars, each partitioned into 5 pieces. Each child gets 8 pieces, which is $8/5$ of a bar.
- ★ Sophisticated answer: 'Each child gets 1 whole bar plus $3/5$ of one — so 1 and $3/5 = 8/5$.'
- ★ The KEY connection of Unit 2: division of whole numbers can give a FRACTIONAL answer.

Problem 10 — Story for $7 \times 2/3$

- Answer: $14/3 = 4$ and $2/3$.
- Strong story: 'There are 7 cups, each $2/3$ full of juice. How much juice in total?' or similar — uses fraction OF something repeated 7 times.
- Weak: story does not actually multiply (e.g., 'There are 7 kids and $2/3$ of them like pizza' — that is not multiplication of these numbers).

Problem 11 — Error analysis: $6 \times 3/8 = 18/48$

- Han multiplied the numerators AND the denominators. The correct rule: numerator $\times$ whole number, denominator unchanged.
- Correct: $6 \times 3/8 = 18/8 = 2$ and $2/8 = 2$ and $1/4$.
- ★ The classic error of treating a whole number as a fraction ($6 = 6/1$ implicitly) and then 'applying' a wrong multiplication rule.
- Strong: 'Han multiplied the denominator by 6 too, but the denominator (8 — what we're cutting INTO) does not change. Only the count of pieces changes.'

— end —