Geometry · Unit 8 · Assessment Guide

Conditional Probability

For the Parent or Teacher — What to look for

Unit 8 is the only unit not about geometry per se. The big ideas: P(A|B) is not the same as P(B|A), and 'independent events' is a precise mathematical condition, NOT just intuition. Look for whether students get the conditional notation right and whether they apply the multiplication rule correctly.

Problem 1 — Spinner and die sample space

- Sample space: 8 x 6 = 48 raw outcomes; 3 x 6 = 18 distinct color-die pairs.
- P(red and 6) = (3/8)(1/6) = 3/48 = 1/16.
- P(yellow and even) = (2/8)(3/6) = 6/48 = 1/8.
- Two events being independent (spinner and die) means multiply the probabilities.

Problem 2 — Card probability with addition rule

- (a) 13/52 = 1/4.
- (b) 12/52 = 3/13. (3 face cards x 4 suits = 12.)
- (c) 3/52. (J, Q, K of hearts.)
- (d) 13/52 + 12/52 - 3/52 = 22/52 = 11/26.
- The Addition Rule subtracts the overlap. A student who forgets the overlap gets the wrong answer.
- Strong students draw a Venn diagram to see the overlap.

Problem 3 — Two-way table conditionals

- P(Math) = 70/200 = 0.35.
- P(9th) = 75/200 = 0.375.
- P(Math AND 9th) = 30/200 = 0.15.
- P(Math | 9th) = 0.15 / 0.375 = 0.40. (Or directly: 30/75 = 0.40.)
- P(9th | Math) = 0.15 / 0.35 ≈ 0.4286. (Or: 30/70.)
- THE diagnostic: P(A | B) is NOT P(B | A). Strong students compute both and notice they're different.
- Common error: dividing by 200 instead of the conditioned subset (75 or 70).

Problem 4 — Independence test from Problem 3

- P(Math) · P(9th) = 0.35 · 0.375 = 0.131.
- P(Math AND 9th) = 0.15.
- 0.131 is not 0.15, so they are NOT independent — being in 9th grade slightly increases the chance of preferring math.
- For perfect independence, P(Math | 9th) would equal P(Math). Currently P(Math | 9th) = 0.40 vs P(Math) = 0.35.
- Diagnostic: whether students recognize that 'almost equal' is not enough for independence — the formula must hold exactly.

Problem 5 — Coin flips

- Independent: yes, because each flip doesn't affect the others.
- P(all heads) = (1/2) cubed = 1/8.
- P(no heads) = (1/2) cubed = 1/8. So P(at least 1 H) = 1 - 1/8 = 7/8.

- The 'at least one' trick (use complement) is a common technique — see if students reach for it or try to enumerate.

Problem 6 — Without replacement

- (a) 5/12.
- (b) 4/11 (one fewer red, one fewer total).
- (c) (5/12)(4/11) = 20/132 = 5/33.
- Why not independent: drawing the first marble REMOVES it from the bag, changing the composition. P(second red) DEPENDS on what was drawn first.
- Strong: explicitly mentions 'the second probability has a different denominator'.

Problem 7 — Venn diagram

- Sports only: 18 - 8 = 10. Music only: 14 - 8 = 6. Both: 8. Neither: 30 - (10 + 6 + 8) = 6.
- Sports but not music: 10.
- Neither: 6.
- P(music | sports) = 8/18 = 4/9.
- Watch for whether students subtract the overlap correctly; many double-count.

Problem 8 — Andre's gambler's fallacy

- Andre is WRONG.
- P(6 on next roll) = 1/6, same as always. Past rolls don't affect future independent events.
- This is the 'Gambler's Fallacy'.
- A classic example of the misconception. Strong students mention 'each roll is independent', so prior outcomes are irrelevant.

Problem 9 — Conditional notation

- Mai is correct.
- P(A | B) = 'probability of A given B' (B is the condition, A is what you're computing).
- Diego mixed up A and B.
- Diagnostic for whether students read the notation correctly.

Problem 10 — Bayesian test problem

- Build a 1000-person table: 20 with disease (2 percent of 1000), 980 without.
- Of 20 with: 19 test positive (0.95 sens), 1 false negative.
- Of 980 without: 98 false positives (0.10), 882 true negatives.
- Total positives: 19 + 98 = 117.
- P(disease | positive) = 19 / 117 ≈ 0.162 = 16.2 percent.
- Surprisingly low! Because the disease is rare AND the test isn't perfect, MOST positives are false positives. This is the famous 'base rate fallacy'.
- This is hard. Strong students set up the table correctly; weak students try to use the conditional formula directly and miss the base rate.

Problem 11 — Student-designed experiment

- Look for: clear question; well-defined sample space; correct identification of independence (or lack thereof); a computation that uses the right tools (multiplication, addition, conditional).
- Strong: an experiment they actually run (e.g., dice, cards, coin flips) with both theoretical and empirical probabilities.
- Weak: vague question; doesn't actually compute anything; uses incorrect formulas.

— *end* —