Geometry • Unit 7 • Assessment Guide

Circles

For the Parent or Teacher — What to look for

Unit 7 covers circle-specific theorems (inscribed angles, tangent-radius, cyclic quadrilaterals) plus the conceptual leap from degrees to radians. Look for whether students understand the WHY behind the inscribed-angle-is-half-central rule, and whether they grasp the DEFINITION of radian.

Problem 1 — Vocabulary

- Chord: a segment whose endpoints lie on the circle.
- Central angle: an angle whose vertex is the CENTER of the circle.
- Inscribed angle: an angle whose vertex is ON the circle and whose sides are chords.
- Minor arc: less than half the circle; Major arc: more than half.
- Common confusion: students think 'chord = diameter' (a diameter is a SPECIAL chord that passes through the center, but most chords don't).

Problem 2 — Inscribed angle theorem

- Central angle = 70 degrees (double the inscribed angle).
- Arc = 70 degrees (same as central angle by definition).
- Inscribed Angle Theorem: The measure of an inscribed angle is HALF the measure of the central angle that subtends the same arc.
- The diagnostic is whether students remember the factor of 2 — and the right direction (inscribed is the SMALLER one).

Problem 3 — Tangent-radius perpendicular

- Angle = 90 degrees.
- Theorem: a tangent line at point P is perpendicular to the radius OP at that point.
- Why: any other line through P would cross the circle at a second point (so it wouldn't be tangent). The radius is the SHORTEST distance from O to the line, and the shortest distance is perpendicular.
- Strong students articulate this geometric reasoning; weak students just cite the theorem.

Problem 4 — Two tangents from external point

- Right angles at P and Q (radius perpendicular to tangent).
- OE = OE (shared/reflexive). OP = OQ (both radii of the same circle).
- Triangles OEP and OEQ are congruent by HL (Hypotenuse-Leg, since they're right triangles).
- Therefore EP = EQ by CPCTC.
- Strong students cite HL correctly; weak students try SSA or just observe 'looks the same'.

Problem 5 — Cyclic quadrilateral

- Angle C = 180 - 80 = 100 degrees. Angle D = 180 - 95 = 85 degrees.
- Why: opposite angles in a cyclic quadrilateral are supplementary because each is HALF the arc it subtends, and the two arcs together make the whole 360 degree circle. So angle A + angle C = (1/2)(arc BCD) + (1/2)(arc BAD) = (1/2)(360) = 180 degrees.
- The 'why' argument is the diagnostic. Strong students draw the arcs; weak students just memorize 'opposite angles supplementary'.

Problem 6 — Circumcenter

- The perpendicular bisector of a side is the locus of points equidistant from the two endpoints of that side. The perpendicular bisectors of two sides intersect at a point P that is equidistant from all three vertices. P also lies on the third perpendicular bisector.
- Construction: build perpendicular bisectors of any two sides; they meet at the circumcenter. Draw a circle through any vertex with center at this point.
- Strong students see that the equidistance argument generalizes; weak students recite 'the three lines meet' without explaining why.

Problem 7 — Incenter vs circumcenter

- Incenter: from angle bisectors; equidistant from SIDES.
- Circumcenter: from perpendicular bisectors of sides; equidistant from VERTICES.
- The diagnostic: do students correctly say SIDES vs VERTICES for distance? They often swap them.
- Strong: explains WHY angle bisector gives equidistance from sides (perpendicular distance interpretation).

Problem 8 — Arc length, sector area, radians

- Arc length = (60/360)·2 pi·12 = (1/6)·24 pi = 4 pi ≈ 12.57.
- Sector area = (60/360)·pi·144 = (1/6)·144 pi = 24 pi ≈ 75.4.
- Radian: arc = r·theta = 12·(pi/3) = 4 pi. Same.
- Why r·theta works: by DEFINITION, one radian is the angle for which arc length = radius. So if theta is in radians, arc = r·theta.
- Strong students articulate the definition; weak just plug into the formula.

Problem 9 — Tyler's chord/diameter confusion

- WRONG. A diameter is a chord that passes through the center, but most chords do not.
- Diameter: a chord through the center; the longest possible chord.
- Chord: ANY segment with both endpoints on the circle.
- Simple definitional check. Look for whether students can give specific, complete definitions.

Problem 10 — Lin's formula verification

- Both give 5 pi. Yes, they agree.
- Why: arc length = (theta/360)·2 pi r in degrees = (theta/180)·pi r. In radians, 180 degrees = pi, so theta_radians = (theta_degrees·pi)/180. Substituting gives the same formula.
- Bigger picture: r·theta_radians IS the underlying formula; the degree version comes from the conversion factor pi/180.
- Strong students see radians as the 'natural' angle measurement; weak see them as an alternative.

Problem 11 — Tire rotation modeling

- Circumference = pi · 26 = 26 pi ≈ 81.68 inches ≈ 6.807 feet.
- Revolutions = 5280/6.807 ≈ 775.6 revolutions.
- Degrees = 775.6 · 360 ≈ 279,222 degrees.
- Radians = 775.6 · 2 pi ≈ 4874 radians.
- A great test of the three angle units. Watch for unit conversion errors (inches vs feet).
- Strong students notice radians = total arc length / radius = (5280·12) / 13 ≈ 4874 — confirms the radian definition.

— end —