Geometry · Unit 6 · Assessment Guide

Coordinate Geometry

For the Parent or Teacher — What to look for

Unit 6 marries algebra with geometry. Watch for: confidence with the distance and slope formulas, ability to recognize parallel/perpendicular conditions, ability to complete the square, and willingness to use coordinates to PROVE (not just verify) geometric facts.

Problem 1 — Coordinate transformations

- (a) Translation: A'(5,-1), B'(9,-1), C'(7,3).
- (b) Reflection: A'(-1,2), B'(-5,2), C'(-3,6).
- (c) Dilation: A'(2,4), B'(10,4), C'(6,12).
- Distance preserved by (a) and (b) — they are rigid. (c) preserves angles only — similar but not congruent.
- Diagnostic: do students articulate that dilation about the origin doubles distances (so it's similar but not congruent)?

Problem 2 — Distance formula

- (a) d = sqrt(25 + 144) = sqrt(169) = 13.
- (b) d = sqrt(144 + 25) = sqrt(169) = 13.
- Both are Pythagorean triples (5-12-13). Look for whether students set up the right legs (horizontal and vertical changes), not whether they remember the formula by rote.

Problem 3 — Circle equations

- (a) (x - 3) squared + (y + 5) squared = 49.
- (b) Radius = sqrt(36 + 64) = 10. So x squared + y squared = 100.
- (c) Complete the square: (x squared - 6x + 9) + (y squared + 4y + 4) = 12 + 9 + 4 = 25. Center (3, -2), radius 5.
- Part (c) is the diagnostic — completing the square in two variables. Students often forget to ADD the same number to the right side.

Problem 4 — Slope and parallel/perpendicular lines

- Slope = (-3 - 5)/(7 - 2) = -8/5.
- Point-slope: y - 5 = -8/5(x - 2).
- Parallel through (1,1): y - 1 = -8/5(x - 1).
- Perpendicular through (1,1): y - 1 = 5/8(x - 1).
- Common error: confusing perpendicular slope (5/8) with opposite slope (8/5). The diagnostic is whether they NEGATE AND RECIPROCATE.

Problem 5 — Why parallel = equal slope, perpendicular = neg recip

- Parallel: a translation preserves slopes (a translation doesn't change orientation). Two lines that translate to each other have the same slope.
- Perpendicular: rotating a line with slope m by 90 degrees about the origin sends (x, y) to (-y, x). A line through origin with direction (a, b) becomes (-b, a). Slope changes from b/a to a/(-b) = -a/b — the negative reciprocal.

● Strong students give the rotation argument or use similar triangles to derive negative reciprocals. Weak: just states the rule.

Problem 6 — Parallelogram proof

● Slope AB = 0. Slope DC = 0 (both horizontal). Parallel.

● Slope AD = (4 - 0)/(3 - 0) = 4/3. Slope BC = (4 - 0)/(8 - 5) = 4/3. Parallel.

● Since both pairs of opposite sides are parallel, ABCD is a parallelogram (by definition).

● Strong: cites the DEFINITION of parallelogram (both pairs of opposite sides parallel) to justify the conclusion.

Problem 7 — Right triangle

● Slope PQ = (4-2)/(4-(-1)) = 2/5. Slope QR = (9-4)/(2-4) = -5/2. Slope PR = (9-2)/(2-(-1)) = 7/3.

● PQ and QR are perpendicular (slopes 2/5 and -5/2 multiply to -1).

● Right angle is at Q.

● Alternative: |PQ| = sqrt(29). |QR| = sqrt(29). |PR| = sqrt(58). And 29 + 29 = 58. Pythagorean check confirms.

● Strong students verify with BOTH methods (slopes and Pythagorean), seeing they agree.

Problem 8 — Partition

● AP:PB = 1:3 means P is 1/4 of the way from A to B. P = (4, 6).

● AP:PB = 3:1 means P is 3/4 of the way from A to B. P = (8, 10).

● Midpoint corresponds to 1:1.

● Common error: confusing 'ratio 1:3' with 'one-third of the way' (it's actually 1/4 of the way).

Problem 9 — Mai's perpendicular error

● Slope 3 and slope -3 multiply to -9, NOT -1. Mai is WRONG.

● Perpendicular to slope 3 is slope -1/3 (negative reciprocal).

● Common confusion of 'opposite' vs 'negative reciprocal'. THE diagnostic.

Problem 10 — Diego's no-circle claim

● Diego is WRONG. The equation IS a circle. Rewrite: x squared + (y squared + 6y + 9) = 9, so x squared + (y+3) squared = 9.

● Center (0, -3), radius 3.

● Diego's error: assuming both x and y terms must appear linearly. The center is just at x = 0 here.

● Strong students complete the square in y only and recognize that the missing x term still gives a circle (centered on the y-axis).

Problem 11 — Coordinate proof of rectangle diagonals

● Place rectangle with vertices (0,0), (a,0), (a,b), (0,b).

● Diagonal 1: from (0,0) to (a,b), length sqrt(a squared + b squared).

● Diagonal 2: from (a,0) to (0,b), length sqrt(a squared + b squared).

● Equal.

● This is a coordinate proof — uses variables (a, b), not specific numbers. The diagnostic is whether students see that variables make this a PROOF (true for ALL rectangles), not just one example.

● Strong: explicitly mentions 'because a and b are arbitrary, this holds for any rectangle.'

— end —