Geometry • Unit 5 • Assessment Guide

Solid Geometry

For the Parent or Teacher — What to look for

Unit 5 is mostly about volume formulas AND scaling. The key conceptual jumps: Cavalieri's Principle (oblique solids have the same volume as right solids with the same base/height), and the k/k squared/k cubed scaling pattern. Watch for procedural errors with formulas vs. conceptual gaps.

Problem 1 — Solids of rotation

- (a) Cylinder (right circular cylinder).
- (b) Right circular cone.
- (c) Sphere.
- This is a 'spatial visualization' check. Students who get (a) and (c) but not (b) often imagine the wrong axis.

Problem 2 — Cube cross-sections

- (a) A square congruent to the face.
- (b) A REGULAR HEXAGON. (Slice perpendicular to the long diagonal through the cube's center.)
- (c) Yes — a pentagonal cross-section is possible (slice the cube so the plane intersects 5 of its faces).
- Strong students sketch the pentagonal or hexagonal cross-section. This is a hard spatial problem; most students will say 'no' to pentagonal — but it IS possible (the slice clips one corner).

Problem 3 — Box scaled by 3

- New dimensions: 12 x 18 x 30.
- Original SA: 2(24 + 40 + 60) = 248 sq. Scaled SA = 248·9 = 2232 sq. Ratio 9 = 3 squared.
- Original V = 240 cubic. Scaled V = 240·27 = 6480 cubic. Ratio 27 = 3 cubed.
- Why: surface area is 2D (each linear dimension x 3, so area x 3·3 = 9). Volume is 3D (x 3·3·3 = 27).
- Weak: just memorizes the rule. Strong: explains using units (square inches vs cubic inches).

Problem 4 — Similar bottles

- Volume ratio 8 means linear ratio cube root of 8 = 2.
- Larger bottle height = 12 · 2 = 24 cm.
- Larger label area = 30 · 2 squared = 120 cm squared.
- The diagnostic is whether students start with the VOLUME ratio and work backward to the linear ratio via cube root.

Problem 5 — Cone, cylinder, sphere

- (a) Cylinder V = pi·25·12 = 300 pi ≈ 942.5 cm cubed.
- (b) Cone V = (1/3)·pi·25·12 = 100 pi ≈ 314.2 cm cubed.
- (c) Sphere V = (4/3)·pi·125 = 500 pi/3 ≈ 523.6 cm cubed.
- Ratio cone : cylinder = 1:3 because of the 1/3 in the cone formula.
- The 'why 1/3' question: it comes from Cavalieri + the fact that 3 congruent triangular pyramids fit together to form a triangular prism (the unit's derivation).

Problem 6 — Oblique cylinder

- Use the PERPENDICULAR height (12), NOT the slant height. V = pi·16·12 = 192 pi ≈ 603.2.
- Cavalieri's Principle says volume only depends on cross-section area (pi r squared at each level) and PERPENDICULAR height — not on slant.
- Common error: students use 15 because it's 'the height of the cylinder'. The diagnostic is whether they realize slant ≠ height.

Problem 7 — Cavalieri's Principle

- Statement: 'If two solids have the same height, and at every level the cross-sections have the same area, the two solids have the same volume.'
- Application: a right cylinder and an oblique cylinder with congruent circular bases at the same perpendicular height have identical circular cross-sections at every level. So volumes are equal.
- Weak: 'Cavalieri said volumes are equal' — restates without principle.
- Strong: articulates 'same area at every height'.

Problem 8 — Iron ball mass

- Volume = (4/3)·pi·216 = 288 pi ≈ 904.8 cm cubed.
- Mass = 904.8 · 7.87 ≈ 7121 g ≈ 7.12 kg.
- Connects geometry to density/physics. A 'practical' problem.
- Watch for unit confusion: cm cubed x g/cm cubed = g, not kg.

Problem 9 — Ice cream cone

- Hemisphere V = (1/2) · (4/3)·pi·27 = 18 pi ≈ 56.5 cm cubed.
- Cone V = (1/3)·pi·9·12 = 36 pi ≈ 113.1 cm cubed.
- Cone holds more than twice the hemisphere — plenty of room.
- Modeling success criterion: clear setup, both volumes computed correctly, sensible comparison.
- Strong students notice that for a cone of height 4r, the cone holds exactly twice the hemisphere of the same radius.

Problem 10 — Andre's tank claim

- WRONG. Doubling all dimensions multiplies volume by 2 cubed = 8, not 2.
- The big idea of the unit. A student who doesn't catch this hasn't internalized the k cubed scaling.
- Strong: gives a concrete example (1x1x1 cube vs 2x2x2 cube; 1 vs 8 cubic units).

Problem 11 — Two cones, same volume

- V_A = (1/3)·pi·16·h_A. V_B = (1/3)·pi·4·h_B.
- Equal: 16·h_A = 4·h_B, so h_B = 4·h_A. Cone B (smaller radius) must be 4 times taller.
- Intuition: cone B is narrower so must be taller to compensate.
- Strong students notice: halving the radius requires 4 times the height to keep volume constant — the radius enters squared in the formula.

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