Geometry · Unit 3 · Assessment Guide

Similarity

For the Parent or Teacher — What to look for

Unit 3 is similarity. The biggest pitfall is treating similar triangles as if they were congruent — keeping the conceptual distinction sharp is the main goal of assessment. Watch how students use proportion and watch for area/length confusion.

Problem 1 — Dilation of a point

- (a) (8, 12). (b) (2, 3). (c) (-4, -6) — negative scale factor through the origin reflects through the origin.
- NONE of (a), (b), (c) preserves distances (distance scales by |k|; only k = 1 or k = -1 preserves length).
- Strong students notice that k = -1 IS rigid (it's a rotation by 180 degrees), but k = 2 and k = 1/2 are not. A student who says 'none' might still be right for a subtle reason — push them to articulate it.

Problem 2 — Dilating a triangle

- A'(0,0), B'(12,0), C'(0,9). Original perimeter 3+4+5=12, image perimeter 9+12+15=36. Ratio 3 (= k).
- Original area = 6. Image area = 54. Ratio 9 (= k squared).
- The 'why' is the diagnostic: area is two-dimensional, so each linear dimension scales by k, giving k·k = k squared for area.
- Volume scaling will reappear in Unit 5 (solid geometry).

Problem 3 — Similar vs Congruent

- (a) Two squares: SIMILAR (always); congruent only if sides match.
- (b) Two equilateral triangles: SIMILAR (always); congruent only if sides match.
- (c) Two rectangles: NEITHER necessarily — they need not be similar (e.g., 1x2 and 1x3).
- (d) Same side length regular hexagons: BOTH (congruent and trivially similar).
- (e) Two right triangles with a 30 degree angle: SIMILAR (they share two angles: 90 and 30, so the third must be 60).
- (c) is the diagnostic: students often think 'same shape category = similar', but rectangles with different aspect ratios are NOT similar.

Problem 4 — AA similarity

- Similar: YES, by AA (two pairs of congruent angles).
- Congruent: NOT NECESSARILY — sides could be any matching proportion.
- Theorem used: Angle-Angle Similarity (AA).
- Strong students explicitly say 'similar but not necessarily congruent — we have no side info.' Weak students collapse the two concepts.

Problem 5 — Using a known ratio

- Scale factor = DE/AB = 9/6 = 1.5.
- DF = 1.5 x 10 = 15. EF = 1.5 x 8 = 12.
- angle D = 36 degrees because similar triangles have congruent corresponding angles.
- Watch the correspondence carefully: students who match the wrong sides (e.g., DE/AC) get wrong answers — flag this.

Problem 6 — Indirect measurement (flagpole)

- Proportion: 6/4 = h/22. So h = 6·22/4 = 33 feet.
- The 'why' question is the diagnostic. The sun's rays being parallel means the angle of elevation is the same for both, so the two triangles have one angle (the right angle, vertical/ground) and another angle (the sun angle) congruent — AA similarity.
- Weak: 'they're proportional' with no reason WHY.

Problem 7 — Altitude in right triangle / Pythagorean proof

- All three triangles share the right angle (90 degrees) and each smaller triangle shares one acute angle with the larger — so by AA, all three are similar.
- From triangle ABC similar to ACD: AB/AC = AC/AD, so AC squared = AB·AD.
- From triangle ABC similar to CBD: AB/BC = BC/BD, so BC squared = AB·BD.
- Adding: AC squared + BC squared = AB·AD + AB·BD = AB·(AD+BD) = AB·AB = AB squared.
- This IS a proof of the Pythagorean Theorem via similarity. Strong students see the elegance: a squared + b squared = c squared emerges naturally.
- Weak: students who can do the algebra but can't articulate why AC squared + BC squared = AB squared is the Pythagorean Theorem.

Problem 8 — Han's area claim

- Han is WRONG. Areas have the ratio of the SQUARES of the side lengths, not the ratio of sides.
- Example: 2x2 square has area 4; 6x6 square has area 36. Side ratio 1:3, area ratio 1:9 = 1:3 squared.
- Common confusion (linked to Problem 2).

Problem 9 — Mai's similarity claim

- Correct: the sides are proportional (6:9 = 8:12 = 10:15 = 2:3).
- Incorrect: she says the angles are different — but if the sides are proportional, the triangles ARE similar (SSS Similarity Theorem), so the angles MUST be equal.
- Actual relationship: these triangles are similar by SSS, with scale factor 3:2 (or 2:3).
- This is the diagnostic question for whether students understand SSS Similarity as well as the more famous AA Similarity.

Problem 10 — Enlarging a photo

- New dimensions: 4·2.5 = 10 inches by 6·2.5 = 15 inches.
- Ink: area scales by 2.5 squared = 6.25, so ink = 12 · 6.25 = 75 sq inches.
- Volume: scales by 2.5 cubed = 15.625.
- Tests the k, k squared, k cubed pattern across all three dimensions. Critical setup for Unit 5.

Problem 11 — Midsegment theorem

- Let D and E be midpoints of sides AB and AC of triangle ABC. Then triangle ADE similar to triangle ABC by SAS similarity (with scale factor 1/2): AD/AB = 1/2, AE/AC = 1/2, and angle A is shared.
- So DE/BC = 1/2, giving DE = (1/2)BC. The angles at D and E correspond to angles at B and C, so by corresponding angles, DE parallel to BC.
- Strong students recognize this as 'shrink the triangle by 1/2 from vertex A'. Weak students try to use congruence and get stuck.

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