Geometry · Unit 2 · Assessment Guide

Congruence

For the Parent or Teacher — What to look for

Unit 2 demands that students JUSTIFY congruence with a specific theorem (SSS, SAS, ASA, AAS) AND identify what is given vs what must be shown. Look for clean reasoning, not just pictures that 'look congruent'.

Problem 1 — CPCTC ordering

- Sides: AB ≅ DE, BC ≅ EF, AC ≅ DF.
- Angles: angle A ≅ angle D, angle B ≅ angle E, angle C ≅ angle F.
- The ordering question is the diagnostic. If triangle ABC ≅ triangle DEF, then triangle ABC ≅ triangle EDF would mean A ↔ E, B ↔ D — usually WRONG. A good answer gives an example showing the correspondence determines which sides/angles are equal.
- Weak: 'we always have to follow the order' — true but no explanation.

Problem 2 — Which postulate applies

- (a) SAS. (b) SSS. (c) ASA. (d) NOT ENOUGH — AAA gives similarity but not congruence. (e) SSA — NOT a congruence postulate (ambiguous case).
- Parts (d) and (e) are the diagnostic: students who write 'AAA' as a congruence postulate are confusing similar triangles with congruent ones — exactly the kind of conflation the unit aims to prevent.

Problem 3 — Triangle ACM ≅ BCM

- Strong proof: 1) AM ≅ BM (M is midpoint, given). 2) angle AMC ≅ angle BMC (both right angles, since CM perp AB). 3) CM ≅ CM (shared / reflexive). 4) Triangle ACM ≅ triangle BCM by SAS.
- The reflexive property (CM ≅ CM) is a common omission. Without it, students don't have three pieces of info, so SAS fails to apply.
- Weak: 'they look congruent because they share a side' — no postulate cited.

Problem 4 — Parallelogram diagonal proof

- Strong proof: AB parallel to CD gives angle BAC ≅ angle DCA (alternate interior angles, transversal AC). Similarly angle BCA ≅ angle DAC. Then AC ≅ AC (reflexive). By ASA, triangle ABC ≅ triangle CDA.
- The diagnostic is whether the student CORRECTLY identifies which angles are alternate interior with which transversal. Many students say 'alternate interior' without specifying the transversal — push them to be explicit.
- Weak: just states 'opposite sides are equal' without proving it (that would be circular).

Problem 5 — Perpendicular bisector theorem

- (a) Outline: take a point P on the perpendicular bisector. Triangle PAM ≅ triangle PBM by SAS (PM shared; right angles equal; AM = BM by definition of bisector). So PA = PB.
- (b) Find perpendicular bisector of AB, then of BC. Their intersection is the circumcenter — equidistant from all three towns.
- This is a 'connecting theorem to context' problem. Strong students give the construction; weaker students just say 'put it in the middle.'

Problem 6 — Parallelogram diagonals bisect each other

- Strong proof: Use the previous problem's result (triangle ABE ≅ triangle CDE) by ASA: angle BAE ≅ angle DCE (alternate interior), AB ≅ CD (opposite sides of parallelogram), angle ABE ≅ angle CDE (alternate interior). Therefore AE ≅ CE and BE ≅ DE by CPCTC.
- The diagnostic: do they USE the previous theorem about opposite sides being congruent, or try to prove from scratch? Building on prior theorems is the unit's main proof skill.

Problem 7 — Quadrilateral T/F

- (a) F — a rhombus has four congruent sides but is not necessarily a square. Counterexample: rhombus with 60/120 degree angles.
- (b) T — opposite sides congruent implies parallelogram.
- (c) F — an isosceles trapezoid has congruent diagonals but is not a rectangle.
- This is a 'distinguish related theorems' question. Strong students give counterexamples; weak students just say T/F with no support.

Problem 8 — Lin's SSA error

- Lin is WRONG. The angle is at A and D, but is NOT included between the two given sides (AB and BC). This is SSA, not SAS, and SSA does not guarantee congruence (the ambiguous case).
- THE diagnostic question of Part E. A student who flags 'angle A is not between AB and BC' has mastered the included-angle subtlety.
- A great counterexample shows two different triangles with the same SSA data.

Problem 9 — Diego's shared side

- Diego IS correct. AC ≅ AD (given), angle CAB ≅ angle DAB (given), AB ≅ AB (reflexive / shared). The shared side counts because reflexive property: a side is congruent to itself.
- Many students dismiss the shared side as 'cheating' — assess whether they understand that reflexive property is a legitimate proof step.
- Weak: thinks the proof is wrong because 'they share that side, so you can't use it twice' — fundamental misunderstanding.

Problem 10 — Isosceles triangle base angles

- Standard proof: drop perpendicular from apex A to base BC at point M. Then AB ≅ AC (given), AM ≅ AM (reflexive), and AM perp BC creates two right angles. Triangle ABM ≅ ACM by HL (or SAS with right angles). Therefore angle B ≅ angle C by CPCTC.
- Alternative: use the angle bisector at A instead of the altitude — gives SAS directly.
- Strong students notice this is a USE of the construction they did in Unit 1 (perpendicular from a point).

Problem 11 — Rhombus diagonals conjecture

- Conjectures should include: diagonals are perpendicular; diagonals bisect each other; diagonals bisect the vertex angles. Any one of these is a strong answer.
- Outline: use triangle congruence (the four small triangles formed by the diagonals are all congruent by SSS or SAS). Then CPCTC gives the perpendicularity (adjacent angles at intersection sum to 180 degrees and are congruent → each is 90 degrees).
- Weak: states the conjecture but cannot sketch the proof — they've memorized the result without internalizing why.

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