# Geometry • Unit 1 • Assessment Guide

*Constructions and Rigid Transformations*

*For the Parent or Teacher — What to look for*

*Unit 1 is the foundation: precision of language, the meaning of 'rigid', and the first introduction to proof writing. Look for evidence the student knows WHY a construction works, not just how to do it.*

## Problem 1 — Vocabulary

- Perpendicular bisector: must include BOTH 'perpendicular to' AND 'passes through midpoint'.
- Rigid transformation: must mention preserving distance (and ideally angle). A student who says 'doesn't change the shape' is close but imprecise — push them to say what is preserved.
- Congruent figures: the formal IM definition is 'one can be mapped to the other by a sequence of rigid transformations'. Accept 'same size and shape' but flag it — by end of Unit 1 the transformation-based definition is the target.
- Weak signal: any definition that relies on measurement ('use a ruler to check').

## Problem 2 — Perpendicular bisector construction

- Steps must specify: (a) circle/arc centered at A with some radius r at least half of AB, (b) same-radius circle/arc centered at B, (c) connect the two intersection points.
- THE diagnostic part is the justification. The two arc intersections P and Q each satisfy PA = PB = r and QA = QB = r. So both are equidistant from A and B, hence both lie on the perpendicular bisector of AB.
- Common weak answer: 'because we used the same radius' — without explaining what equal radius IMPLIES (equal distance from each endpoint).
- Strong: students who explicitly invoke 'a point equidistant from the endpoints lies on the perpendicular bisector.'

## Problem 3 — Equilateral triangle construction

- AC = AB because C is on the circle centered at A with radius AB.
- BC = AB because C is on the circle centered at B with radius AB.
- The third part is where many students stumble: WHY equilateral, not just isosceles? Because all three sides equal AB, not merely two. The student should reference the definition of equilateral (all three sides equal) and show all three have measure AB.
- Weak answer: 'the triangle looks equilateral'.

## Problem 4 — Coordinate transformations

- (a) (-1, 6). (b) (2, -5). (c) (-5, 2). The rotation is the most commonly missed; watch for sign confusion.
- The follow-up is the diagnostic: rigid transformations preserve distance and angle. A student should not just say 'shape stays the same' but should mention that distances between any two points and the measures of any angles are preserved.
- Strong: links 'rigid' to congruence of pre-image and image.

## Problem 5 — Reflection without a grid

- (a) Yes; reflections are rigid so preserve distance.
- (b) Yes; by definition of reflection, the line of reflection is the perpendicular bisector of the segment from any point to its image.
- (c) C' = C; points ON the line of reflection are fixed.

- Part (b) is THE diagnostic for whether the student has internalized the formal off-the-grid definition of reflection. A student who says 'I'm not sure' is still thinking only in terms of grid moves.

### Problem 6 — Two methods for constructing a square

- Method 1 most students will give: construct perpendiculars at A and B to segment AB, then mark off length AB on each perpendicular.
- Method 2 strong alternative: construct circles to locate vertices via rotation by 90 degrees, or use the diagonals.
- Real test: are the methods genuinely different, or just two ways to draw the same construction? Two perpendiculars (one at each endpoint) counts as ONE method; perpendiculars vs. rotations counts as TWO methods.
- The efficiency reflection should mention number of compass moves or steps.

### Problem 7 — Tyler vs Mai (definition of rigid)

- Mai is correct. A dilation by scale factor 2 doubles distances, so it is NOT rigid.
- Strong: Tyler's mistake is conflating 'same shape' (similarity) with 'same size and shape' (congruence/rigid). The student should clearly distinguish.
- Weak: 'Tyler is wrong because dilations aren't rigid' — restates without explaining.

### Problem 8 — Andre using a ruler

- Andre's result might happen to be correct but it is not a valid compass-and-straightedge construction.
- The diagnostic answer: constructions are about LOGICAL guarantees from compass/straightedge axioms. A ruler measurement is an approximation that depends on the ruler's accuracy. Compass moves give equal distances by definition.
- Weak: 'rulers aren't allowed' — restates the rule without explaining the principle.
- Strong: connects to the bigger idea that constructions yield exact equalities, supporting later proofs.

### Problem 9 — Symmetries of regular polygons

- (a) Equilateral triangle: 3 reflections + 3 rotations (including identity) = 6 total. Accept students who count 2 non-identity rotations + 3 reflections = 5; flag whether they count the identity.
- (b) Non-square rectangle: 2 reflections + 1 non-identity rotation (180 degrees) + identity = 4 (the Klein 4-group).
- (c) Regular hexagon: 6 reflections + 6 rotations = 12 total.
- Pattern: regular n-gon has 2n symmetries (n rotations + n reflections). A student who notices this without being told demonstrates strong abstract thinking.

### Problem 10 — Triangle Angle Sum proof

- Construction: through C, draw a line parallel to AB. This creates angles at C congruent to angles A and B by alternate interior angles.
- The narrative should connect: (1) the line through C parallel to AB; (2) alternate interior angles for the transversal AC and the transversal BC; (3) the three angles at C lie along a straight line, summing to 180 degrees.
- Common weak proof: 'I measured the angles and they added to 180.' Measurement is evidence, not proof.
- Strong proof reads as a connected argument; weak proofs are bullet points without logical glue.

### Problem 11 — Open construction (rhombus from two circles)

- Construction: draw circle 1 centered at A through B; draw circle 2 centered at B through A. The two intersection points P and Q satisfy AP = AB = AQ = BP = BQ.

● The rhombus is APBQ (with diagonals AB and PQ).

● Strong students will justify with the same logic as Problem 2: equal radii give equal distances.

● Very strong: they notice AP = BP = AQ = BQ = AB, so all four sides of APBQ are congruent — a rhombus by definition.

● Weak: draws something that looks like two circles but cannot articulate why a rhombus forms.

*— end —*