Algebra 2 · Unit 7 · Assessment Guide

Trigonometric Functions

For the Parent or Teacher — What to look for

Unit 7 is about trigonometric FUNCTIONS, not just ratios. The most diagnostic moments are whether students see sine/cosine as outputs of a point-on-the-unit-circle process — not as memorized magic. Watch for the radian/degree confusion; it's the single most common mistake.

Problem 1 — Unit-circle exact values

- (a) $\cos(0) = 1$, $\sin(0) = 0$.
- (b) $\cos(\pi/2) = 0$, $\sin(\pi/2) = 1$.
- (c) $\cos(\pi) = -1$, $\sin(\pi) = 0$.
- (d) $\cos(\pi/4) = \sin(\pi/4) = \sqrt{2}/2$.
- (e) $\cos(\pi/3) = 1/2$, $\sin(\pi/3) = \sqrt{3}/2$.
- (f) $\cos(\pi/6) = \sqrt{3}/2$, $\sin(\pi/6) = 1/2$.
- ★ The Unit 7 fluency test. A student who has a unit circle reference card and uses it for every value gets full marks. A student who can derive these from the special right triangles (30-60-90 and 45-45-90) shows deeper structural understanding.

Problem 2 — Pythagorean Identity application

- $\sin^2(\theta) = 9/25$, so $\cos^2(\theta) = 16/25$, so $\cos(\theta) = \pm 4/5$.
- Quadrant II $\Rightarrow \cos(\theta) < 0$, so $\cos(\theta) = -4/5$.
- $\tan(\theta) = (3/5) / (-4/5) = -3/4$.
- ★ THE diagnostic: do they pick the correct sign for $\cos(\theta)$ based on the quadrant? A student who takes only the principal root forgets that cos can be negative.

Problem 3 — Beyond the first cycle

- (a) $\cos(-\pi/3) = \cos(\pi/3) = 1/2$ (cosine is EVEN).
- (b) $\sin(7\pi/6) = -\sin(\pi/6) = -1/2$ (Quadrant III, ref angle $\pi/6$).
- (c) $13\pi/4 - 2\pi = 5\pi/4$ (still > 2π no — wait, $13\pi/4 = 3.25\pi$, subtract $2\pi = 8\pi/4$ to get $5\pi/4$). $\cos(5\pi/4) = -\sqrt{2}/2$.
- (d) Strong: "After turning a full circle (2π), you're back at the same point on the unit circle, so sin and cos repeat."
- ★ Symmetry properties (even/odd) carry over from Unit 6 — connection point.

Problem 4 — Standard sin/cos graphs

- Both have amplitude 1, period 2π, midline $y = 0$.
- $\cos(x) = \sin(x + \pi/2)$. Cosine is sine shifted LEFT by $\pi/2$.
- ★ Strong students draw KEY POINTS at $0, \pi/2, \pi, 3\pi/2, 2\pi$. Weak students sketch generic 'wavy lines' without specific values.

Problem 5 — Trig transformations

- Amplitude: 3.
- Period: $2\pi/2 = \pi$.
- Midline: $y = 1$.

- Phase shift: $\pi/4$ to the RIGHT.
- Max: $1 + 3 = 4$. Min: $1 - 3 = -2$.
- ★ Watch for the period error: dividing 2π by the FREQUENCY (b) gives period, not multiplying. Most common mistake: "period = $2\pi \cdot 2 = 4\pi$."

Problem 6 — Ferris wheel model

- Height function (starting at bottom): $h(t) = 25 - 20\cdot\cos(2\pi\cdot t/30) = 25 - 20\cdot\cos(\pi t/15)$.
- Amplitude: 20. Midline: $y = 25$. Period: 30.
- Why MINUS cos? At $t=0$, the rider is at the bottom ($h = 5$), so we need $h(0) = 25 - 20 = 5$. ✓
- $h(10) = 25 - 20\cdot\cos(2\pi/3) = 25 - 20\cdot(-1/2) = 25 + 10 = 35$ feet.
- ★ The hardest part: building the model from words. Most students get the amplitude and midline; few correctly handle the 'starts at the bottom' constraint by negating cos.

Problem 7 — Tangent asymptotes

- $\tan(x)$ is undefined where $\cos(x) = 0$: $x = \pi/2, 3\pi/2, 5\pi/2, \ldots, -\pi/2, \ldots$
- Vertical asymptotes there because dividing by zero → unbounded.
- Period of tan = π (not 2π). Both sin and cos flip sign over π, so their RATIO returns to the same value — tan has half the period.
- ★ The 'why π not 2π' question separates students who understand sin/cos as a ratio from those who memorize.

Problem 8 — Radians vs. degrees

- Diego put '30' into a calculator set to RADIANS mode. $\sin(30 \text{ rad}) \approx -0.988$ because 30 radians = many full rotations + a bit.
- Correct: either type $\sin(\pi/6)$, or switch the calculator to DEGREES mode and type $\sin(30°)$.
- ★ THE most common Unit 7 mistake. A diagnostic moment for whether students understand that radians and degrees are two different units for the SAME angle.

Problem 9 — Lin's incorrect identity

- $\sin(\pi/3) = \sqrt{3}/2 \approx 0.866$. $2\cdot\sin(\pi/6) = 2\cdot(1/2) = 1$.
- $0.866 \neq 1$, so the formula is FALSE.
- Correct: $\sin(2x) = 2\cdot\sin(x)\cdot\cos(x)$ (double-angle formula — typically not introduced until later, but the test by substitution catches the error).
- ★ The general lesson: sine is NOT linear, so $\sin(2x) \neq 2\cdot\sin(x)$. Strong students articulate that trig functions are highly nonlinear.

Problem 10 — Trig in nature

- ★ Open-ended. Strong answers connect periodic phenomena to trig:
- Sound waves: pressure oscillates sinusoidally.
- Tides: driven by Earth/Moon rotation (circular motion → sinusoidal projection).
- AC current: voltage as sin function of time.
- Planetary motion: positions on a circle project to sinusoidal coordinates.
- ★ The unifying property: anything that REPEATS at a constant rate can be modeled with sine/cosine, because that's exactly what the unit circle traces out.

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