Algebra 2 · Unit 6 · Assessment Guide

Transformations of Functions

For the Parent or Teacher — What to look for

Unit 6 unifies what students have learned about transformations across all function types. The hardest single idea: horizontal transformations (involving the INPUT) work OPPOSITE to what intuition suggests, because they act on the input before the function does.

Problem 1 — Translation of a parabola

- (a) Up 4. Vertex (0, 4).
- (b) Right 3. Vertex (3, 0).
- (c) Left 2, down 5. Vertex (−2, −5).
- (d) ★ THE deepest probe of the section. Strong: "$f(x - 3)$ takes the input x, subtracts 3, then squares. So when x = 3, the function reads (0) — what it used to read at 0. The graph shifts RIGHT because each input now hits a point that was previously to its left."
- Weak: "because minus shifts right" — memorized rule with no understanding.

Problem 2 — Even/odd classification

- (a) $f(-x) = (-x)^4 - 3(-x)^2 = x^4 - 3x^2 = f(x)$. EVEN.
- (b) $g(-x) = -x^3 + x = -(x^3 - x) = -g(x)$. ODD.
- (c) $h(-x) = x^2 - x \neq h(x)$, and $\neq -h(x)$. NEITHER.
- (d) Even: graph is symmetric about the y-axis. Odd: graph has rotational symmetry about the origin (180°). Neither: no special symmetry.
- ★ Strong students give both the algebraic CHECK and the geometric interpretation.

Problem 3 — Scaling probe

- (a) Vertical stretch by 2. (Heights doubled.)
- (b) Horizontal compression by factor 1/2. (Graph squeezed toward y-axis.)
- (c) Vertical compression by 1/3.
- (d) Horizontal STRETCH by 3.
- (e) ★ THE diagnostic question. Strong: "Vertical scaling multiplies the OUTPUT directly. Horizontal scaling acts on the INPUT — so $f(2x)$ reads 2x where it used to read x, meaning you reach the same height twice as fast (compressed). The reciprocal effect comes from solving 2x = old-x: x = old-x/2."
- Weak: states the rule without the reciprocal insight.

Problem 4 — Build transformations

- (a) $g(x) = f(x + 2) + 5$.
- (b) $h(x) = -3 \cdot f(x)$.
- (c) $p(x) = f(-x)$.
- (d) $q(x) = f(2x) + 1$.
- ★ Watch for sign errors in (a) — students who write $f(x - 2)$ intend a left shift but get a right shift.

Problem 5 — Sketching with transformations

- $g(x) = -|x - 4| + 3$.

- Start with $|x|$ (V-shape at origin). Shift RIGHT 4 (vertex at (4, 0)). Reflect over x-axis (now inverted V at (4, 0)). Shift UP 3 (vertex at (4, 3)).
- Final vertex: (4, 3). Opens DOWNWARD.
- ★ Strong students apply transformations in the correct order: scaling/reflecting around the INPUT first, then vertical reflection, then vertical shift.

Problem 6 — Diver as transformed parabola

- $h(t) = -16(t^2 - 1.5t) + 6 = -16(t^2 - 1.5t + 0.5625) + 9 + 6 = -16(t - 0.75)^2 + 15$.
- Vertex: (0.75, 15). Diver reaches MAX height 15 ft at 0.75 seconds after launch.
- (c) Transformations of t^2: reflect over x-axis, vertical stretch by 16, shift right 0.75, shift up 15.
- ★ This connects Unit 6 transformations back to Algebra 1 quadratics — a deeper-than-procedure question.

Problem 7 — Mai's RIGHT/LEFT confusion

- WRONG. $f(x + 5)$ shifts LEFT 5, not right.
- Test: $f(x) = x^2$, $f(0) = 0$. Under $f(x + 5)$, the input 0 becomes $(0 + 5) = 5$, so $f(0 + 5) = f(5) = 25$ — the value that used to live at $x = 5$ now lives at $x = 0$. Graph shifted LEFT.
- ★ This is the most common Unit 6 error. The 'opposite' behavior of horizontal shifts is the central diagnostic.

Problem 8 — Han's premature odd conclusion

- $f(2) = 9$. $f(-2) = -8 + 1 = -7$. $-f(2) = -9$. So $f(-2) \neq -f(2)$. NEITHER.
- ★ Adding 1 BREAKS the odd symmetry of x^3. Strong students see that any odd function plus a nonzero constant is NEITHER (unless the constant is zero).
- Connect to: $f(x) = x^3$ has rotational symmetry about origin; adding 1 shifts the center of symmetry to (0, 1), which violates the strict 'odd' definition (which requires symmetry through origin).

Problem 9 — Tracking a point through transformations

- Method 1 (plug in): Need x such that $x - 1 = 4$, so $x = 5$. Then $g(5) = 2 \cdot f(4) + 5 = 2 \cdot 9 + 5 = 23$.
- Method 2 (trace transformations): Start at (4, 9). $x - 1$ in the input means shift RIGHT 1: now at (5, 9). Multiply f by 2: now at (5, 18). Add 5: now at (5, 23). Same answer ✓.
- ★ Both methods MUST yield (5, 23). Strong students cross-check.

Problem 10 — Universality of transformations

- ★ Open-ended. Strong answers note:
- Transformations don't depend on which specific function f you have — the rules apply universally.
- Studying them once lets you handle quadratic, cubic, exponential, log, radical, and (in Unit 7) trigonometric all with the same toolkit.
- ★ Best: connects this to Unit 7 trigonometric transformations (which is the explicit pedagogical motivation per the unit narrative).

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