Algebra 2 · Unit 4 · Assessment Guide

Complex Numbers and Rational Exponents

For the Parent or Teacher — What to look for

Unit 4 stretches the number system in two directions: rational exponents (filling gaps in the reals) and complex numbers (extending beyond reals). Diagnostic errors cluster around: forgetting ± on square roots, mishandling $i^2 = -1$, and treating $\sqrt{-9}$ as a real number.

Problem 1 — Rational exponents

- (a) $8^{(1/3)} = \sqrt[3]{8} = 2$.
- (b) $16^{(3/4)} = (\sqrt[4]{16})^3 = 2^3 = 8$.
- (c) $25^{(-1/2)} = 1/\sqrt{25} = 1/5$.
- (d) ★ Strong: "$a^{(1/n)}$ means the number that, raised to the n, gives a — exactly what an nth root is. Exponent rules force $(a^{(1/n)})^n = a^1 = a$."
- Weak: just states the rule without reasoning.

Problem 2 — Squares vs. square roots

- (a) $x = \pm 7$ (TWO solutions).
- (b) $x = 49$ (ONE solution).
- (c) ★ THE diagnostic. Strong: "$\sqrt{x}$ denotes the PRINCIPAL (non-negative) square root by convention, so $\sqrt{x} = 7$ has only one solution. But $x^2 = 49$ allows BOTH +7 and −7 since both square to 49."
- If students say (a) has only $x=7$, they've lost the ± again — a recurring Unit 2/4 issue.

Problem 3 — Radical equation with extraneous solution

- Squaring: $x + 5 = (x - 1)^2 = x^2 - 2x + 1$. $\Rightarrow x^2 - 3x - 4 = 0 \Rightarrow (x - 4)(x + 1) = 0 \Rightarrow x = 4$ or $x = -1$.
- Check $x = 4$: $\sqrt{9} = 3 = 4 - 1 = 3$. ✓
- Check $x = -1$: $\sqrt{4} = 2$; but $x - 1 = -2$. So $2 \neq -2$. EXTRANEOUS.
- ★ Strong: "Squaring turns '$a = b$' into '$a^2 = b^2$', which is ALSO true when $a = -b$. So we pick up solutions where the two sides have opposite signs — those don't satisfy the original."
- This is the same root cause as Unit 3 extraneous solutions.

Problem 4 — Powers of i

- $i^3 = i^2 \cdot i = -i$. $i^4 = (i^2)^2 = 1$. $i^5 = i$. $i^6 = i^2 = -1$.
- (b) Strong: "$x^2 = -1$ has no real solution, so we invent i. Once i exists, all quadratics are solvable."
- (c) i^{50}: $50 = 4\cdot 12 + 2$, so $i^{50} = i^2 = -1$.
- ★ The diagnostic is whether they realize i has a 4-cycle: i, −1, −i, 1, i, −1, ...

Problem 5 — Complex arithmetic

- $z + w = 4 - 2i$. $z - w = 2 + 6i$.
- $z\cdot w = (3+2i)(1-4i) = 3 - 12i + 2i - 8i^2 = 3 - 10i - 8(-1) = 3 - 10i + 8 = 11 - 10i$.
- $z^2 = (3+2i)^2 = 9 + 12i + 4i^2 = 9 + 12i - 4 = 5 + 12i$.
- ★ Most common error on $z\cdot w$: forgetting $i^2 = -1$ and writing $-8i^2 = -8$. The whole point of complex multiplication is that i^2 flips signs.

Problem 6 — Two methods for $x^2 + 4x + 13 = 0$

- Complete the square: $(x + 2)^2 + 9 = 0 \Rightarrow (x + 2)^2 = -9 \Rightarrow x + 2 = \pm 3i \Rightarrow x = -2 \pm 3i$.
- Quadratic formula: $x = (-4 \pm \sqrt{16 - 52})/2 = (-4 \pm \sqrt{-36})/2 = (-4 \pm 6i)/2 = -2 \pm 3i$.
- ★ Both methods MUST give the same answer; that's the cross-check. A student who gets different answers should look for arithmetic errors.

Problem 7 — Discriminant analysis

- (a) $D = 36 - 36 = 0$. ONE real (repeated) solution: $x = 3$.
- (b) $D = 9 + 40 = 49 > 0$. TWO real solutions.
- (c) $D = 4 - 20 = -16 < 0$. TWO COMPLEX solutions.
- (d) ★ Strong: "the quadratic formula has $\sqrt{D}$ in it; if $D > 0$ we get two real values, if $D = 0$ the ± collapses to one, if $D < 0$ we get an imaginary part — two complex conjugate solutions."
- This connects to the Unit 4 unit narrative perfectly.

Problem 8 — Han's $\sqrt{-9}$ claim

- Han is WRONG on multiple levels. $\sqrt{-9}$ is not a real number at all; it equals $3i$ (the principal complex square root).
- Even if we tried to extend √ to negatives without i, the convention would still give the PRINCIPAL (positive imaginary) root: $3i$, not $-3i$.
- ★ Strong students articulate that the symbol √ denotes one specific value, even for negatives: $\sqrt{-9} = 3i$.

Problem 9 — Mai's no-solution radical

- Substituting $x = 16$: $\sqrt{16} = 4$, but the equation says $\sqrt{x} = -4$. Since $4 \neq -4$, $x = 16$ is NOT a solution.
- $\sqrt{x} = -4$ has NO solutions, because $\sqrt{x}$ is always ≥ 0 (by convention).
- ★ This is the cleanest possible example of how squaring introduces extraneous solutions: the original equation has no solutions, but squaring ($x = 16$) appears to manufacture one out of thin air.
- Connect this back to Problem 3 and to Unit 3 extraneous solutions.

Problem 10 — Conjugate product

- $(a + bi)(a - bi) = a^2 - (bi)^2 = a^2 - b^2 \cdot i^2 = a^2 - b^2 \cdot (-1) = a^2 + b^2$.
- ★ Always real, and always non-negative.
- (b) Strong: "To divide complex numbers, multiply numerator and denominator by the conjugate of the denominator. The denominator becomes real, which is what we want for standard form $a + bi$."
- This is groundwork for advanced complex arithmetic and matters in the unit's 'Working Backward' optional section.

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