Algebra 2 · Unit 3 · Assessment Guide

Rational Functions and Equations

For the Parent or Teacher — What to look for

Unit 3 is about structure of rational expressions. Most diagnostic errors hide in the SAME PLACE: forgetting that x-values that make a denominator zero are excluded from the domain. Extraneous solutions are simply the algebraic shadow of that fact.

Problem 1 — Basic rational function f(x) = (x+3)/(x−2)

- Domain: all real x except x = 2 (denominator zero).
- Vertical asymptote: x = 2.
- Horizontal asymptote: y = 1 (degrees of numerator and denominator equal; ratio of leading coefs = 1/1).
- x-intercept: x = −3 (numerator zero). y-intercept: f(0) = 3/(−2) = −3/2.
- ★ The diagnostic for (c) is whether they explain the rule: equal degrees ⇒ HA is ratio of leading coefs; numerator smaller ⇒ HA is y=0; numerator larger ⇒ no HA.

Problem 2 — End behavior by degree comparison

- (a) HA: y = 2 (equal degrees; ratio of leading coefs 4/2).
- (b) HA: y = 0 (numerator degree LESS than denominator).
- (c) NO horizontal asymptote — numerator degree (3) > denominator degree (1). There is an oblique/polynomial asymptote.
- ★ This is the classic Unit 3 rule. Most students get (a) and (b) but fall apart on (c). If they say 'HA is infinity' or '$y = x^2$' (the actual oblique trend), accept the latter with explanation.

Problem 3 — Hole vs vertical asymptote

- r(x) = (x−2)(x+2)/(x−2). Simplifies to r(x) = x + 2 for x ≠ 2.
- At x = 2 there is a HOLE, not a vertical asymptote — the factor cancels.
- r(3) = 5. The function 'wants' the value 4 at x = 2 (limit) but is undefined there.
- ★ THE most diagnostic question. A student who declares x = 2 is a vertical asymptote without simplifying is applying a rule without thinking. Strong students always factor and cancel first.

Problem 4 — 1/(x−3) = 5/(x+1)

- Cross-multiply: x + 1 = 5(x − 3) ⇒ x + 1 = 5x − 15 ⇒ 16 = 4x ⇒ x = 4.
- Check: 1/(4−3) = 1; 5/(4+1) = 1. ✓
- ★ Both x = 3 and x = −1 must be excluded from the domain. The solution x = 4 is in the domain, so it's valid. Strong students articulate this domain-check explicitly.

Problem 5 — Extraneous solution

- Multiply both sides by (x − 2): x = 2 + (x − 2) ⇒ x = x. True for all x! BUT x = 2 was excluded from the domain.
- Actual solution set: all real x EXCEPT x = 2.
- Alternatively if a student combines and gets x = 2 directly (e.g., subtracting (x−2)/(x−2) gives 0=0), they need to recognize x=2 is extraneous.
- ★ This is the deepest extraneous-solution moment of the unit. The lesson: multiplying by (x − 2) erases the very restriction that makes the equation different from x = x.

Problem 6 — Average cost

- $A(n) = (5000 + 20n)/n = 5000/n + 20$.
- $A(100) = \$70$/item. $A(10{,}000) = \$20.50$/item.
- As $n \to \infty$, $A(n) \to \$20$. Horizontal asymptote $y = 20$.
- In context: as you make more items, the fixed cost is spread thinner, but you can never push the average BELOW the per-item cost of $20.
- ★ Strong students explicitly connect the asymptote $y = 20$ to the real-world floor of "$20 of variable cost per item."

Problem 7 — Polynomial identity

- Expanding: $(x - 1)(x^2 + x + 1) = x^3 + x^2 + x - x^2 - x - 1 = x^3 - 1$. ✓ Identity.
- It's true for ALL real values (and even complex values) of x.
- Using $x = 3$: $27 - 1 = 26$, factored as $(3 - 1)(9 + 3 + 1) = 2 \cdot 13 = 26$. ✓
- ★ The diagnostic: do they understand 'identity' means 'true for all x' vs. 'equation' which holds for specific x? Strong students distinguish.

Problem 8 — Geometric sum

- $a = 1$, $r = 3$, $n = 7$.
- $S = (1 - 3^7)/(1 - 3) = (1 - 2187)/(-2) = (-2186)/(-2) = 1093$.
- Verification: $1+3+9+27+81+243+729 = 1093$. ✓
- ★ Strong students notice the structure: $3^7 = 2187$, so $S = (3^7 - 1)/2$. This pattern recurs throughout Algebra 2 and connects to Unit 1.

Problem 9 — Cross-multiplication scrutiny

- Lin set up $3 = 2x$ (multiplying both sides of $3/x = 2$ by x). Then $x = 3/2 = 1.5$. Check: $3/1.5 = 2$ ✓.
- So she is RIGHT, but her LANGUAGE is confused — '$3 \cdot 1 = 2 \cdot x$' is hand-wavy. The clear setup: multiply both sides by x.
- ★ The diagnostic: students who say she's wrong without checking are pattern-matching against "cross-multiply means multiply diagonals". Strong students verify by substitution.

Problem 10 — Why extraneous solutions arise

- ★ Open-ended. Strong: "Multiplying both sides by an expression containing x can multiply by ZERO when x makes that expression zero. Multiplying by zero turns any equation into 0 = 0, which is true but meaningless. So those x-values become apparent solutions even though they are not."
- Weak: "sometimes the algebra adds extra answers" — true but missing the mechanism.
- ★ This question is the whole conceptual payoff of Unit 3.

— end —