Algebra 2 · Unit 2 · Assessment Guide

Polynomial Functions

For the Parent or Teacher — What to look for

Unit 2 builds polynomial thinking around the structure of factored form. Most diagnostic errors hide in multiplicity, in confusing zeros with x-intercepts vs y-intercept, and in losing track of the ± when undoing a square.

Problem 1 — Identifying polynomials

- (a) Polynomial, degree 4, leading coef 3.
- (b) NOT a polynomial — $\sqrt{x}$ is $x^{(1/2)}$, a non-integer exponent.
- (c) Polynomial. Degree 3, leading coef 1.
- (d) NOT a polynomial — x^{-2} has a negative exponent.
- Watch for: students who say (c) is not a polynomial because "it's in factored form." Form does not change identity — it just looks different.

Problem 2 — Factored form ↔ graph

- Zeros: $x = -3$ (mult 1), $x = 1$ (mult 2), $x = 4$ (mult 1).
- Degree 4, leading coefficient 1 when expanded.
- End behavior: both ends UP (degree 4, positive leading coef).
- Graph: crosses x-axis at −3 and 4, TOUCHES and bounces at $x = 1$.
- y-intercept: $p(0) = (3)(-1)^2(-4) = -12$.
- ★ Most diagnostic: do they show the BOUNCE at $x = 1$? If they draw a straight crossing, they don't yet apply multiplicity to shape.

Problem 3 — Multiplicity behavior

- Mult 1: crosses transversally (line-like).
- Mult 2: TOUCHES and bounces (parabola-like, never crosses).
- Mult 3: crosses but with an inflection point (S-curve-like, flatter near zero).
- ★ Strong: mentions that $(x-a)^2$ is always ≥ 0, so the sign of the polynomial doesn't change as you cross $x = a$; hence touch, not cross.
- Weak: memorized rule with no reasoning.

Problem 4 — Two methods for $2x^3 - 8x = 0$

- Factor: $2x(x^2 - 4) = 2x(x - 2)(x + 2) = 0$, giving $x = 0, 2, -2$.
- Graph: find x-intercepts of $y = 2x^3 - 8x$ — same three values.
- Most students will find factoring faster here because it's a tidy cubic. Graphing is slower BUT works when factoring is hard.
- ★ A strong student notes: "graphing wins when the polynomial doesn't factor nicely or when I want a quick visual sanity check."

Problem 5 — Polynomial division + Remainder Theorem

- (a) Quotient: $x^2 - 5x + 6$. Remainder: 0.
- (b) Remainder 0 ⇒ $(x - 1)$ IS a factor.
- (c) $p(x) = (x - 1)(x - 2)(x - 3)$. Zeros: 1, 2, 3.

- Watch for: students who try to factor the quotient $x^2 - 5x + 6$ incorrectly. It factors as $(x - 2)(x - 3)$.
- ★ The diagnostic: do they see that $p(x)$ had to be divisible by $(x - 1)$ because $p(1) = 1 - 6 + 11 - 6 = 0$? That's the Remainder Theorem in action.

Problem 6 — Remainder Theorem statement

- Strong: "If $p(2) = 0$, then $(x - 2)$ is a factor of $p(x)$." — this is the Factor Theorem.
- Even better: connects to the Remainder Theorem: dividing $p(x)$ by $(x - 2)$ gives a remainder equal to $p(2) = 0$.
- Weak: "$x = 2$ is a zero" — true but doesn't use the language of factors.

Problem 7 — Build polynomial from zeros + point

- Factored form: $p(x) = a(x + 2)(x - 3)^2$.
- Use (0, 36): $36 = a(2)(-3)^2 = 18a$, so $a = 2$.
- Final: $p(x) = 2(x + 2)(x - 3)^2$.
- ★ The slot for 'a' is the diagnostic. Many students write the factors correctly but forget that a stretching constant is needed to match the point (0, 36).

Problem 8 — Mai's missing root

- Mai is WRONG — she lost the negative root. $x^2 = 9 \Rightarrow x = \pm 3$.
- ★ THE most common Algebra 2 error of the year, and a setup for Unit 4 (square roots ⇒ two solutions to $x^2 = k$).
- Strong: "Taking a square root of both sides gives ±, because both 3^2 and $(-3)^2$ equal 9."
- Weak: "she should have written +/−" without saying why.

Problem 9 — Different factors, different polynomials

- $(x + 1)(x - 2) = x^2 - x - 2$. Zeros at −1 and 2.
- $(x - 1)(x + 2) = x^2 + x - 2$. Zeros at 1 and −2.
- Diego is WRONG — different signs in the factors give different polynomials with different zeros.
- ★ The conceptual point: the SIGNS inside the parentheses matter. The factor $(x - a)$ gives zero at $x = +a$, not $x = -a$.

Problem 10 — End behavior reflection

- ★ Open-ended; look for the key idea: for $|x|$ very large, the leading term DOMINATES because higher powers grow much faster than lower powers.
- Strong: gives a numerical example — for $p(x) = x^4 - 100x^2$, $p(10) = 10000 - 10000 = 0$, but $p(100) = 10^8 - 10^6 \approx 10^8$. The $-100x^2$ is negligible.
- Weak: states a rule without explaining why.

— end —