Algebra 2 • Unit 1 • Assessment Guide

Sequences and Functions

For the Parent or Teacher — What to look for

Unit 1 reintroduces functions through sequences. Watch for whether students can move flexibly between recursive and explicit forms — that flexibility, more than computational correctness, is the diagnostic signal of understanding.

Problem 1 — Classification + d/r

- (a) arithmetic, d=3. (b) geometric, r=3. (c) NEITHER (square numbers — differences grow). (d) geometric, r=1/2.
- ★ The diagnostic is (c). A student who calls it arithmetic because "the differences exist" hasn't internalized that arithmetic requires CONSTANT differences. Strong students notice the differences 3, 5, 7, 9 — themselves an arithmetic sequence.
- (e) Strong: "check if successive DIFFERENCES are constant (arithmetic) or successive RATIOS are constant (geometric)." Weak: "if it grows, it's arithmetic."

Problem 2 — Recursive ⇔ Explicit

- (a) 3, 7, 11, 15, 19.
- (b) $f(n) = 3 + 4(n-1)$ or equivalently $4n - 1$. Watch the classic off-by-one: students writing $f(n) = 3 + 4n$ start the sequence at 7, not 3.
- (c) $f(50) = 199$.
- (d) ★ Strong: "explicit lets me jump to term 50 directly; recursive would require 49 additions." This is the WHOLE POINT of having two forms.
- If a student picks "recursive" as easier here, they don't yet see the trade-off — flag for follow-up.

Problem 3 — Geometric sequence

- (a) $g(1) = 16$, $g(n) = (1/2)\cdot g(n-1)$ for $n \geq 2$.
- (b) $g(n) = 16 \cdot (1/2)^{(n-1)}$. Common error: $g(n) = 16\cdot(1/2)^n$ which gives 8 at n=1, off by one.
- (c) $g(8) = 16 \cdot (1/2)^7 = 16/128 = 1/8$.
- If they get a decimal like 0.125 without explaining, ask them to write it as a fraction — fraction fluency is a Unit 1 expectation.

Problem 4 — Modeling tiles

- (a) $\text{Tiles}(n) = 4 + 3(n-1) = 3n + 1$. Same off-by-one issue as Problem 2.
- (b) ★ THE diagnostic for the unit. Domain should be POSITIVE INTEGERS ($n = 1, 2, 3, \ldots$). NOT $n = 0$ (no Step 0 in the problem). NOT $n = 1.5$ (you can't have half a step).
- Weak answer: "all real numbers" — they're treating it like a continuous function.
- (c) The pattern could continue forever in principle, but the model is a sequence because the input is restricted to integers (step numbers).

Problem 5 — Han's error

- Han is WRONG. The sequence is geometric (ratio 2), not arithmetic.
- Han's misconception: he thinks "arithmetic" just means "following a rule." In mathematics it specifically means CONSTANT DIFFERENCE.
- Strong: "the DIFFERENCES are 3, 6, 12 — not constant — but the RATIOS are all 2. It's geometric."

Problem 6 — Lin vs. Diego

- Lin: $f(1) = 5 + 3(1) = 8$. Wrong — should be 5.
- Diego: $f(1) = 2 + 3(1) = 5$. Correct! $f(n) = 2 + 3n$.
- Equivalent correct form: $f(n) = 5 + 3(n-1)$.
- ★ Lin's error is the classic off-by-one: she didn't shift for the n=1 start. Watch how the student explains it — "she forgot to subtract 1 from n" is good; "she added wrong" misses the structural issue.

Problem 7 — Multiple representations

- Table: 20.25, 13.5, 9, 6, 4.
- Recursive: $h(1) = 20.25$, $h(n) = (2/3)\cdot h(n-1)$.
- Explicit: $h(n) = 20.25 \cdot (2/3)^{(n-1)}$.
- Watch: do they keep 20.25 as 81/4 to make the algebra cleaner, or as a decimal? Fluent students often switch to fractions because $(2/3)^k$ stays exact.
- ★ Strong students notice that $20.25 = 81/4$ and $81 = 3^4$, which gives a tidy form: $h(n) = 3^{(5-n)} / 4$. Bonus signal.

Problem 8 — Doubling savings (sum of geometric sequence)

- Terms: 1, 2, 4, 8, 16, 32, 64, 128, 256, 512.
- Sum = $2^{10} - 1 = 1023$ dollars. (This is the formula $1 + 2 + 4 + \ldots + 2^{(n-1)} = 2^n - 1$.)
- ★ Strong students recognize the doubling pattern and the "power of 2 minus 1" trick from the binary-counting story.
- (d) The 30-day total: $2^{30} - 1 \approx \$1.07$ BILLION. The diagnostic is whether students realize their intuition will be wildly wrong — exponential growth defies linear intuition. This is a Unit 1 reflection moment.

Problem 9 — Arithmetic vs. geometric growth

- Geometric (with $r > 1$) always overtakes arithmetic eventually, regardless of starting values.
- Strong example: $f(n) = 100 + 5n$ vs $g(n) = 2\cdot 1.5^n$. At n=1, f=105 and g=3. By $n\approx 25$, g overtakes.
- ★ The fact they need to grasp: "exponential beats linear for large enough n" — this is foundational for Unit 5.
- Weak: gives one example without numerical evidence of crossover.

Problem 10 — Create-your-own (sequence appropriate)

- Strong: "Number of leaves on a tree branch where each level has double the leaves." Domain: positive integers (levels).
- Strong: "Population census taken every 5 years." Domain: discrete time points.
- Weak: "speed of a car over time" — that's CONTINUOUS, not a sequence. A flag that they don't grasp domain restrictions.
- ★ The justification must mention discrete inputs.

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