Algebra 1 · Unit 8 · Assessment Guide

Quadratic Equations

For the Parent or Teacher — What to look for

Unit 8 brings all of Algebra 1 to a head: students must MOVE FLEXIBLY between methods (factoring, square roots, completing the square, quadratic formula) and connect each algebraic solution back to a graph. Watch closely for the choice-of-method, the lost-solution trap (dividing by x), and the discriminant as the structural diagnostic.

Problem 1 — Reasoning-based solves

- (a) $x = \pm 7$. (b) $x - 3 = \pm 4$, so $x = 7$ or $x = -1$. (c) $x^2 = 25$, $x = \pm 5$. (d) $(x + 1)^2 = 25$, $x + 1 = \pm 5$, so $x = 4$ or $x = -6$.
- (e) ★ The KEY lesson: $x^2 = 49$ means $x \cdot x = 49$; both +7 and −7 work. Students who write only $x = 7$ have forgotten that squaring loses sign info.
- Watch: any answer with a SINGLE value to a square-root equation is a red flag.

Problem 2 — Factor + Zero Product

- (a) $(x - 3)(x - 5) = 0$, $x = 3$ or 5.
- (b) $x(x + 4) = 0$, $x = 0$ or -4.
- (c) $(x - 4)(x + 4) = 0$, $x = \pm 4$.
- (d) ★ Han LOST the solution $x = 0$. Dividing by x assumes $x \neq 0$; if $x = 0$ is actually a solution, you've thrown it away. Correct approach: factor x out — $x(x - 4) = 0$, giving $x = 0$ or $x = 4$.
- ★ THE classic algebra trap of this unit.

Problem 3 — Complete the square on $x^2 + 6x - 7$

- $x^2 + 6x = 7$. Add $(6/2)^2 = 9$: $x^2 + 6x + 9 = 16$. $(x + 3)^2 = 16$. $x + 3 = \pm 4$. $x = 1$ or $x = -7$.
- Strong: clean isolation, correct identification of $(b/2)^2$, proper ± step.
- Watch: forgetting to add 9 to the OTHER side; or using $b/2 + b/2$ instead of $(b/2)^2$.

Problem 4 — Rewrite in vertex form

- $x^2 - 10x + 18 = (x^2 - 10x + 25) - 25 + 18 = (x - 5)^2 - 7$.
- Vertex form: $(x - 5)^2 - 7$. Vertex: $(5, -7)$.
- ★ Connects Unit 7 (vertex form) with Unit 8 (completing the square): SAME PROCEDURE, different framing.

Problem 5 — Quadratic formula on $2x^2 - 5x + 1 = 0$

- $x = (5 \pm \sqrt{(25 - 8)})/4 = (5 \pm \sqrt{17})/4$. $x \approx (5 + 4.12)/4 \approx 2.28$ or $x \approx (5 - 4.12)/4 \approx 0.22$.
- Watch: sign errors substituting b; arithmetic with $b^2 - 4ac$; division by 2a not by a alone.

Problem 6 — Discriminant predictions

- (a) $36 - 20 = 16$. Discriminant > 0 → 2 real solutions.
- (b) $16 - 16 = 0$. Discriminant = 0 → exactly 1 real solution (double root).
- (c) $1 - 4 = -3$. Discriminant < 0 → NO real solutions.
- (d) Graphically: discriminant > 0 → parabola crosses x-axis twice; = 0 → touches once; < 0 → never crosses.

- ★ The discriminant connects algebra (number of roots) and geometry (intercepts) — the structural capstone.

Problem 7 — Same equation, two methods

- $x^2 - 6x + 5 = 0 \rightarrow (x - 1)(x - 5) = 0 \rightarrow x = 1$ or 5.
- Quadratic formula: $x = (6 \pm \sqrt{(36 - 20)})/2 = (6 \pm 4)/2 = 5$ or 1.
- Strong: both methods give same answer; understands they're EQUIVALENT, not competing.
- ★ Method-flexibility is the diagnostic; speed often matters in choice.

Problem 8 — Which method?

- (a) Square root ($x = \pm 9$). Simplest case.
- (b) Factoring: $(x - 3)(x - 4) = 0$; integer zeros, easy.
- (c) Quadratic formula (or completing the square): doesn't factor over integers.
- (d) Quadratic formula (discriminant $25 - 84 < 0$, no real solutions).
- ★ A skilled algebra-1 student SELECTS method based on the equation's structure; this is the unit's capstone skill.

Problem 9 — Rectangular garden

- (a) $w(w + 3) = 70 \rightarrow w^2 + 3w - 70 = 0$.
- (b) Factor: $(w + 10)(w - 7) = 0$, so $w = 7$ or $w = -10$. Or quadratic formula gives same.
- (c) $w = 7$ ft (length 10 ft). Reject $w = -10$ because width can't be negative.
- ★ The CONTEXTUAL filtering of algebraic solutions is the modeling diagnostic.

Problem 10 — Ball hits ground

- $-16t^2 + 48t + 4 = 0 \rightarrow$ divide by 4: $-4t^2 + 12t + 1 = 0$, or $4t^2 - 12t - 1 = 0$.
- $t = (12 \pm \sqrt{(144 + 16)})/8 = (12 \pm \sqrt{160})/8 = (12 \pm 4\sqrt{10})/8 = (3 \pm \sqrt{10})/2$.
- $\sqrt{10} \approx 3.16$, so $t \approx (3 + 3.16)/2 \approx 3.08$ sec. Reject the negative root.
- Watch: students may compute correctly but forget to reject the negative time.

Problem 11 — Force one solution: $x^2 - 4x + c = 0$

- (a) Discriminant $b^2 - 4ac = 16 - 4c = 0 \rightarrow c = 4$.
- (b) Equation becomes $x^2 - 4x + 4 = 0$, i.e., $(x - 2)^2 = 0$, so $x = 2$.
- ★ Reverse-engineering from discriminant tests deep understanding.

Problem 12 — Sums/products of rational and irrational

- (a) Always rational (closure of rationals under addition).
- (b) Could be either: $\sqrt{2} + (-\sqrt{2}) = 0$ (rational); $\sqrt{2} + \sqrt{3}$ (irrational).
- (c) Always irrational. Strong: proof sketch — assume rational, derive contradiction.
- (d) E.g., $x^2 - 2 = 0$ has $x = \pm\sqrt{2}$ (irrational).
- ★ The closure properties of number sets — the conceptual high-water mark of the unit.

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