Algebra 1 · Unit 7 · Assessment Guide

Introduction to Quadratic Functions

For the Parent or Teacher — What to look for

Unit 7 introduces the third major family of functions (after linear and exponential): quadratics. Students should see HOW quadratic patterns differ from linear and exponential ones, and how each FORM (standard, factored, vertex) reveals a different feature of the graph. Watch for understanding of the connection between equation form and graph feature.

Problem 1 — First/second differences

- (a) A: first differences are 3, 3, 3, 3 — CONSTANT → linear. B: first differences are 1, 3, 5, 7 — NOT constant.
- (b) B: SECOND differences are 2, 2, 2 — CONSTANT.
- (c) Sequences with constant SECOND differences are quadratic. (And constant first differences = linear; constant ratios = exponential.)
- ★ The structural diagnostic — same approach as Unit 6, one level up.

Problem 2 — Projectile $h(t) = -16t^2 + 32t + 5$

- (a) $h(0) = 5$: ball starts at 5 ft (platform height).
- (b) $h(1) = -16 + 32 + 5 = 21$. $h(2) = -64 + 64 + 5 = 5$. The ball rose, then fell back.
- (c) Quadratic formula on $-16t^2 + 32t + 5 = 0$: $t = (-32 \pm \sqrt{(1024 + 320)})/(-32) = (-32 \pm \sqrt{1344})/(-32)$. $\sqrt{1344} \approx 36.66$. Positive root: $t \approx 2.15$ sec.
- (d) Max at vertex: $t = -b/(2a) = -32/(-32) = 1$ sec. $h(1) = 21$ ft.
- (e) Domain: $0 \le t \le 2.15$ (ball exists from launch to impact). ★ Modeling literacy.

Problem 3 — Three forms, same function

- (a) Expand factored: $(x - 1)(x - 3) = x^2 - 4x + 3$ ✓. Expand vertex: $(x - 2)^2 - 1 = x^2 - 4x + 4 - 1 = x^2 - 4x + 3$ ✓.
- (b) Zeros: $x = 1$ and $x = 3$.
- (c) Vertex: $(2, -1)$.
- (d) y-int: $f(0) = 3$.
- (e) ★ THE diagnostic of this unit: Standard form → y-intercept (constant term). Factored form → x-intercepts (zeros). Vertex form → vertex (max or min). Strong students articulate all three.

Problem 4 — Graph $(x + 1)(x - 3)$

- (a) Zeros: $x = -1$ and $x = 3$.
- (b) Vertex $x = (-1 + 3)/2 = 1$.
- (c) $g(1) = (2)(-2) = -4$.
- (d) $g(0) = (1)(-3) = -3$.
- Sketch: upward parabola, vertex $(1, -4)$, through $(-1, 0)$, $(0, -3)$, $(3, 0)$.
- Watch for: forgetting the vertex y-coord (just plotting $(1, 0)$).

Problem 5 — How a affects $f(x) = ax^2$

- All have vertex $(0, 0)$.
- (a) Standard parabola.

- (b) Narrower (steeper).
- (c) Wider (shallower).
- (d) Opens DOWN (vertex is now a maximum).
- (e) ★ Sign of a controls UP vs. DOWN. Magnitude of a controls steepness — large $|a|$ → narrow; small $|a|$ → wide.

Problem 6 — L-shape pattern

- (a) Total depends on careful reading: $2n + 1$ in base + n on side = $3n + 1$. Linear.
- (b) LINEAR ($3n + 1$), not quadratic. Strong students confirm by checking first differences.
- (c) Step 10 = 31; step 100 = 301.
- ★ This problem checks whether students can recognize when a pattern that LOOKS geometric is actually linear, not quadratic.

Problem 7 — Quadratic vs. exponential

- $f(1) = 1$, $g(1) = 2$. $f(5) = 25$, $g(5) = 32$. $f(10) = 100$, $g(10) = 1024$. $f(20) = 400$, $g(20) \approx 1{,}048{,}576$.
- g overtakes f around $x = 4$ or 5 (and stays ahead forever).
- ★ The 'eventually exponential wins' insight — extending Unit 6's takeaway.

Problem 8 — $g(x) = -2(x - 3)^2 + 4$

- (a) Vertex (3, 4). Vertex form $(x - h)^2 + k$ → vertex is (h, k).
- (b) Opens DOWN ($a = -2 < 0$).
- (c) Narrower than f ($|a| = 2 > 1$).
- (d) Has a MAX at $y = 4$.
- (e) Order: starts with x^2; shift RIGHT 3 (subtract inside); vertical stretch by 2; REFLECT over x-axis; shift UP 4.
- ★ Order matters here; strong students articulate horizontal-first or inside-vs-outside.

Problem 9 — Error analysis

- (a) $(x + 3)^2 = x^2 + 6x + 9$. Andre forgot the middle term. ★ The single most common algebra-1 mistake. Strong students explain using $(x + 3)(x + 3)$ distribution.
- (b) Lin is wrong — vertex form is $(x - h)^2 + k$, so $f(x) = (x - 5)^2 + 2$ has $h = 5$, vertex (5, 2). The MINUS sign in '$(x - 5)$' makes h POSITIVE. Lin grabbed the wrong sign.
- Both are diagnostic — sign errors of two different kinds.

Problem 10 — Farmer's rectangle (open modeling)

- (a) Three sides of fencing: $2x + y = 100$, so $y = 100 - 2x$. Area $A = xy = x(100 - 2x) = 100x - 2x^2$.
- (b) Max at $x = -b/(2a) = -100/(-4) = 25$. Max area = $25 \cdot 50 = 1250$ sq ft.
- (c) Domain: $0 < x < 50$.
- (d) Vertex (25, 1250). Downward parabola.
- ★ A modeling capstone — students must set up the optimization, recognize the quadratic, and find the max.

— end —