Algebra 1 · Unit 6 · Assessment Guide

Introduction to Exponential Functions

For the Parent or Teacher — What to look for

Unit 6 introduces a fundamentally NEW type of function — one whose CONSTANT RATIO (not constant difference) of outputs governs its behavior. Watch whether students grasp this distinction, can interpret growth factors vs. growth rates, and can resist 'linearizing' exponential thinking (especially around compound interest).

Problem 1 — P linear vs. Q exponential

- (a) P adds 3 each step. Q multiplies by 2.
- (b) $P(x) = 10 + 3x$. $Q(x) = 10 \cdot 2^x$.
- (c) $P(10) = 40$. $Q(10) = 10 \cdot 1024 = 10{,}240$. Vastly different.
- (d) ★ THE central insight of the unit: Q's RATE OF CHANGE is itself growing (it's a CONSTANT MULTIPLIER on a constantly growing base). P's rate of change is fixed. Eventually, multiplicative growth always overtakes additive growth.
- Watch: students who say "Q is steeper from the start" miss the conceptual point.

Problem 2 — Bacteria doubling

- (a) $B(t) = 50 \cdot 2^t$.
- (b) $B(5) = 50 \cdot 32 = 1600$.
- (c) Solve $50 \cdot 2^t > 10{,}000 \rightarrow 2^t > 200$. $2^7 = 128$, $2^8 = 256$. So about 8 hours.
- (d) Real bacteria run out of food, space, oxygen. Exponential models GROW WITHOUT BOUND — a critical caveat. ★ Modeling literacy.

Problem 3 — Drug decay

- (a) $D(t) = 80 \cdot (0.8)^t$.
- (b) DECAY FACTOR = 0.8. DECAY RATE = 20% (i.e., 0.20). Strong students articulate: rate is the percent CHANGE; factor is what you MULTIPLY by.
- (c) $D(4) = 80 \cdot (0.8)^4 = 80 \cdot 0.4096 \approx 32.8$ mg. $D(10) = 80 \cdot 0.1074 \approx 8.6$ mg.
- (d) Mathematically, $D(t)$ approaches 0 but NEVER REACHES 0 (asymptote at 0). In real life, eventually so dilute it's effectively zero, but the exponential model never hits exactly 0.

Problem 4 — $f(x) = 3 \cdot 2^x$

- (a) y-int = $f(0) = 3 \cdot 2^0 = 3$. The COEFFICIENT (3) IS the y-intercept for functions of this form.
- (b) $f(1) = 6$, $f(2) = 12$, $f(3) = 24$.
- (c) $f(-1) = 3 \cdot 1/2 = 1.5$. $f(-2) = 3 \cdot 1/4 = 0.75$. Negative exponent means "divide by the base" repeatedly.
- (d) Graph: smoothly curving up; passes through (0, 3), (1, 6), (2, 12), etc. Approaches 0 as x decreases.
- ★ Negative exponents are a Unit 6 milestone — watch for students who write $f(-1) = -6$.

Problem 5 — Match equations to descriptions

- (a) Exponential DECAY (base $1/2 < 1$), y-int 10, decreasing.
- (b) Exponential GROWTH (base $3 > 1$), y-int 5, increasing.
- (c) Exponential DECAY (base $0.95 < 1$), y-int 200, slow decrease.
- (d) Constant function = 1 (base 1).

Problem 6 — Compound interest at 6%

- (a) Year 1: \$1060. Year 3: $1000(1.06)^3 \approx$ \$1191.02.
- (b) $A(t) = 1000(1.06)^t$.
- (c) Actual: $1000(1.06)^{10} \approx$ \$1790.85, so interest ≈ \$790.85 — NOT \$600.
- (d) ★ THE diagnostic of compounding. Han added 6% to the ORIGINAL each year (simple interest). Compound interest charges 6% on the GROWING balance. The concept Han misses: 'interest on interest.'

Problem 7 — Successive discounts

- (a) Total discount is NOT 30%. After 20% off: 0.8 of original. Then 10% off that: $0.8 \times 0.9 = 0.72$. Total discount = 28%, not 30%.
- (b) Final price = \$72.
- ★ The multiplicative-vs-additive distinction — same lesson as compounding, different setting.

Problem 8 — Constant-ratio detective

- (a) $18/12 = 1.5$. $27/18 = 1.5$. $40.5/27 = 1.5$.
- (b) Constant ratio → EXPONENTIAL function with growth factor 1.5.
- (c) $y = 8 \cdot (1.5)^x$. (Check: $y(1) = 12$ ✓.)
- ★ The structural diagnostic: SAME RATIO = exponential. (Compare to: same DIFFERENCE = linear.)

Problem 9 — Exponent rule errors

- (a) Priya is WRONG. 5^x means $5 \cdot 5 \cdot 5 \cdot \ldots \cdot 5$ (x times), not $5 \cdot x$. Counter: $5^3 = 125$, not 15.
- (b) Mai is WRONG. Same base: ADD exponents. $2^3 \cdot 2^4 = 2^7 = 128$, not 2^{12}.
- (c) Diego is WRONG. Power of a power: MULTIPLY exponents. $(3^2)^4 = 3^8$.
- ★ The three classic exponent-rule errors. Each is a misapplication; explaining the right rule explicitly is the goal.

Problem 10 — \$1000/month vs. doubling pennies

- (a) Scenario A: \$30,000 in 30 days. Scenario B: total = $2^{30} - 1$ cents ≈ \$10,737,418 — wildly larger. Doubling wins easily.
- (b) Strong: "Exponential growth starts slow but accelerates dramatically; even tiny amounts compound to huge numbers." Weak: just picks A because "\$1000/month sounds like more."
- ★ The most memorable exponential-growth lesson; check for the 'aha.'

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