Algebra 1 · Unit 5 · Assessment Guide

Functions

For the Parent or Teacher — What to look for

Unit 5 is the conceptual heart of Algebra 1. Students transition from manipulating expressions to thinking of functions as objects: with notation, graphs, domains, and inverses. Watch the difference between students who can DO algebra with f(x) and students who understand what f IS.

Problem 1 — Function or not?

- (a) Function — linear; each x maps to one y.
- (b) NOT a function — $x = y^2$ means $y = \pm\sqrt{x}$; two outputs.
- (c) Function — vertical-line test passes.
- (d) Function — same input gives same output, which is allowed.
- (e) NOT a function — same input (2) gives two different outputs (5 and 7).
- ★ (d) vs. (e) is the diagnostic — the rule is "one output per input," not "different outputs per input."

Problem 2 — Evaluate $f(x) = 2x^2 - 3x + 1$

- (a) $f(0) = 1$.   (b) $f(-2) = 8 + 6 + 1 = 15$.
- (c) $f(a+1) = 2(a+1)^2 - 3(a+1) + 1 = 2(a^2 + 2a + 1) - 3a - 3 + 1 = 2a^2 + 4a + 2 - 3a - 2 = 2a^2 + a$.
- (d) $f(x) = 1 \rightarrow 2x^2 - 3x = 0 \rightarrow x(2x - 3) = 0 \rightarrow x = 0$ or $x = 3/2$.
- Common errors: (c) forgetting to square the binomial fully; (d) dividing by x and losing the solution $x = 0$.

Problem 3 — Coffee cooling graph

- (a) $g(0) = 90$°C — initial temperature.
- (b) Decreasing — coffee cools.
- (c) Coffee approaches room temperature near 22°C.
- (d) NOT linear — rate of cooling is FASTER initially and slows over time (Newton's law of cooling). Strong students may describe this as 'concave-up' decreasing.
- Look for: connection between the SHAPE of the graph and the physical situation.

Problem 4 — Average rate of change

- (a) $(16 - 4)/(3 - 1) = 12/2 = 6$.
- (b) $(64 - 16)/(7 - 3) = 48/4 = 12$.
- (c) Different rates → f is NOT LINEAR. (These values match f(x) = 2^x, but students don't need to identify that.)
- (d) For a linear function, average rate of change is the same over EVERY interval (it equals the slope).
- ★ (c)/(d) connects rate of change to function type — the conceptual bridge of the unit.

Problem 5 — Taxi cost function

- (a) $C(m) = 3.50 + 2.25m$.
- (b) Domain: $m \geq 0$ (can't drive negative miles), and likely some upper bound (the city has finite size). Strong students name both bounds.
- (c) Range: $C \geq 3.50$ (everyone pays at least the flat fee), up to some max.

- (d) Different function — rounding up would be a STEP function (piecewise constant), not linear. Strong students recognize this is a discrete-output function.
- ★ Domain/range in CONTEXT is the diagnostic — not just mathematical possibility but what makes sense.

Problem 6 — Piecewise h(x)

- (a) $h(-4) = 2(-4)+1 = -7$; $h(0) = 0^2 = 0$; $h(2) = 4$; $h(3) = 9$; $h(5) = 12 - 5 = 7$.
- (b) At $x = 0$: left limit (from $2x+1$) is 1, right value is 0 — NOT continuous (jump of 1). At $x = 3$: left value is 9, right limit (from $12 - x$) is 9 — CONTINUOUS.
- (c) Domain: all real numbers. Range: includes a range of values from each piece.
- ★ (b) is the diagnostic — testing continuity at the seams is the technical core of piecewise functions.

Problem 7 — Absolute value transformations

- (a) g is shifted RIGHT 3 and UP 2 from f.
- (b) f-vertex: (0, 0). g-vertex: (3, 2).
- (c) Outside vs. inside is the diagnostic. +2 OUTSIDE happens AFTER computing $|x - 3|$, so it changes the OUTPUT (y-direction). −3 INSIDE happens BEFORE the absolute value, so it changes the INPUT — and the RIGHT shift (not left) happens because we need $x = 3$ to make $x - 3 = 0$, the vertex.
- ★ This 'inside-vs-outside' rule generalizes to all function transformations — a key understanding for Algebra 2 and beyond.

Problem 8 — Inverse of f(x) = 3x − 6

- (a) Swap and solve: $y = 3x - 6 \rightarrow x = 3y - 6 \rightarrow y = (x + 6)/3$. So f-inverse(x) = $(x + 6)/3$.
- (b) f-inverse(5) = 11/3. $f(11/3) = 3(11/3) - 6 = 11 - 6 = 5$. ✓
- (c) f converts \$ to €; f-inverse converts € back to \$. The inverse 'undoes' the conversion.
- ★ The contextual interpretation is the diagnostic of true understanding.

Problem 9 — Han's and Diego's claims

- (a) Han's claim $f(x + h) = f(x) + h$ is generally FALSE — only true for f linear with slope 1 (or constant 0). Counter: $f(x) = x^2$; $f(2 + 1) = 9$, $f(2) + 1 = 5$.
- (b) Diego's notation IS standard — $(f \cdot g)(x) = f(x) \cdot g(x)$. This is the DEFINITION of the product of two functions. Always true.
- Watch: students may dismiss both without examples; the discipline of EXAMPLE-TESTING is the lesson.

Problem 10 — Parabola vertex (2, −1)

- (a) Decreasing on $(-\infty, 2)$; increasing on $(2, \infty)$.
- (b) Minimum value −1, at $x = 2$.
- (c) No max because the parabola opens upward — y can grow without bound.
- Strong: connects 'opens upward' with 'has min but no max.'

Problem 11 — Sketch a function with given properties

- Their sketch should: cross x-axis at $x = 0$ and $x = 3$; be above the axis on $(-\infty, 0)$ and $(3, \infty)$; have a peak at (around) $x = -1$; have a dip at (around) $x = 2$.
- ★ This is the inverse direction — from PROPERTIES back to a GRAPH — the deepest test of how the student conceives a function.

— end —