Algebra 1 · Unit 2 · Assessment Guide

Linear Equations and Systems

For the Parent or Teacher — What to look for

Unit 2 builds the central tools of Algebra 1: writing, manipulating, and solving linear equations and systems. The goal is not procedural fluency alone — it's whether the student understands WHY each step is valid and can interpret solutions in context. Track this distinction throughout.

Problem 1 — Party planner modeling

- (a) Variables with UNITS: p = pizzas, b = soda bottles. Equation: $12p + 3b = 180$.
- (b) $12(8) + 3(28) = 96 + 84 = 180$. ✓ Yes.
- (c) (15, 0) satisfies the equation ($12 \cdot 15 = 180$), but "buy 15 pizzas and no drinks" might be silly in context. ★ The gap between mathematical solutions and contextual solutions is the diagnostic — strong students raise it.
- (d) Many possible: (10, 20), (5, 40), (12, 12), etc. Watch for negatives or fractions — neither makes sense here.

Problem 2 — Are A and B equivalent?

- Expand A: $3x - 12 = 2x + 1$, so $x = 13$. Plug $x = 13$ into B: $3(13) = 2(13) + 13 \to 39 = 39$. ✓
- Both have the same solution $x = 13$, so they are equivalent.
- Strong: shows the algebraic transformation. Weak: just plugs in numbers and gets lucky.

Problem 3 — Valid moves

- (a) Valid — subtracting same value from both sides.
- (b) INVALID — multiplying by 0 destroys information; $0 = 0$ is true for ALL x, not just the solution.
- (c) INVALID — dividing by x is dividing by an unknown; you would LOSE the solution $x = 0$.
- (d) Valid — distributing is rewriting, doesn't change solutions.
- ★ (b) and (c) are the diagnostic — a student who calls (c) valid will eventually lose solutions on quadratics.

Problem 4 — Rearranging $F = (9/5)C + 32$

- (a) $C = (5/9)(F - 32)$.
- (b) $C = (5/9)(50 - 32) = (5/9)(18) = 10$. So 50°F = 10°C.
- (c) ★ The PURPOSE of rearranging: efficiency for repeated computations + clarity of meaning. Strong: "You only do the algebra once, then can plug in any F."

Problem 5 — Solve $5x - 3(2 - x) = 4x + 14$

- $5x - 6 + 3x = 4x + 14 \to 8x - 6 = 4x + 14 \to 4x = 20 \to x = 5$.
- Check: $5(5) - 3(2 - 5) = 25 - 3(-3) = 25 + 9 = 34$. RHS: $4(5) + 14 = 34$. ✓
- Classic mistakes: distributing the −3 as $-3 \cdot 2 - 3 \cdot x$ (which gives $-6 - 3x$) instead of $-6 + 3x$. WATCH SIGNS.

Problem 6 — $4x + 2y = 12$

- (a) $y = -2x + 6$.
- (b) x-int: set $y = 0$: $4x = 12$, $x = 3 \to (3, 0)$. y-int: set $x = 0$: $2y = 12$, $y = 6 \to (0, 6)$.
- (c) Line from (0, 6) through (3, 0); slope is −2.

• Watch for: forgetting to divide BOTH terms by 2 (gives $y = -2x + 12$ — wrong intercept).

Problem 7 — Line through (2, 11), slope 3

• Point-slope: $y - 11 = 3(x - 2)$. Slope-intercept: $y = 3x + 5$.

• Strong: shows the algebraic step from one to the other (distribute then add 11). Connects the two forms — same line, two ways to describe it.

Problem 8 — System by two methods

• Solution: $x = 4$, $y = 3$.

• Substitution: from eq 1, $y = 11 - 2x$. Substitute: $3x - 2(11 - 2x) = 6 \rightarrow 7x = 28 \rightarrow x = 4, y = 3$.

• Elimination: multiply eq 1 by 2: $4x + 2y = 22$. Add to $3x - 2y = 6$: $7x = 28 \rightarrow x = 4, y = 3$.

• ★ Doing BOTH and getting the same answer builds confidence in the methods. If they get different answers, that's the bug to find.

Problem 9 — How many solutions?

• (a) NO solution — same slope (2) but different y-intercepts (parallel lines).

• (b) INFINITELY MANY — second equation is just 2× the first; they're the same line.

• (c) ONE solution — different slopes (−3 and 1), so lines intersect once.

• ★ Strong reasoning: connects equation structure to graphical meaning without solving.

Problem 10 — Han's elimination

• Han is CORRECT! $x = 7$, $y = 3$ satisfies both equations ($7 + 3 = 10$, $7 - 3 = 4$).

• (b) The KEY conceptual question. Strong: "If $x + y = 10$ and $x - y = 4$, both true for the same (x, y). Adding equal quantities to equal quantities preserves equality. So $(x + y) + (x - y) = 10 + 4 \rightarrow 2x = 14$ still holds."

• Weak: "because it works." No reasoning about why.

Problem 11 — Bus vs. van rental

• System: $200 + 0.5m = 50 + 0.8m \rightarrow 150 = 0.3m \rightarrow m = 500$ miles.

• (b) At 600 miles: Bus = 200 + 300 = \$500; Van = 50 + 480 = \$530. Bus is cheaper by \$30.

• (c) The intersection is the BREAK-EVEN point: below it, the lower-flat-rate option (van) wins; above it, the bus.

• ★ The interpretation step is the diagnostic — students who can solve but not explain the meaning haven't fully internalized the model.

Problem 12 — Make a system with solution (4, −3)

• Endless possibilities. Simplest: $x = 4$, $y = -3$ (trivial but valid). More interesting: $x + y = 1$ and $x - y = 7$.

• Second system: $y = -3$ and $x = 4$. Or $2x - y = 11$ and $x + 2y = -2$.

• INFINITELY MANY systems share any given solution.

• ★ This question reveals whether a student sees a system as "any pair of equations both satisfied by the same point" — i.e., understands the meaning, not just the procedure.

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