Accelerated 7 · Unit 8 · Assessment Guide

Pythagorean Theorem and Irrational Numbers

For the Parent or Teacher — What to look for

Unit 8 ties together geometry, algebra, and number sense via the Pythagorean Theorem and irrational numbers. The deep ideas: (1) area of a tilted square gives side length via square root; (2) the theorem relates leg lengths to hypotenuse via squares; (3) some numbers (sqrt(2), pi) can't be written as fractions — they're irrational. This is a major preparation for Algebra 1 and Geometry.

Problem 1 — Square area to side

- Area 49 -> side 7. Area 30 -> side sqrt(30) (exact). Decimal ≈ 5.48 is acceptable but inexact.
- Strong: leaves answer as sqrt(30) — recognizing this is the EXACT value.
- Weak: writes 5.5 and stops, not appreciating the difference between exact and approximate.

Problem 2 — Draw squares with given areas

- Area 16: 4 by 4 axis-aligned square.
- Area 5: TILTED square — vertices like (0,0), (2,1), (1,3), (-1,2). Side length sqrt(5).
- Area 25: 5 by 5 axis-aligned OR tilted with legs 3,4 (side sqrt(3^2+4^2)=sqrt(25)=5).
- Tilted squares unlock the visual proof of Pythagoras.

Problem 3 — Find missing side

- (a) c^2 = 36 + 64 = 100, c = 10.
- (b) a^2 + 25 = 169, a^2 = 144, a = 12.
- (c) c^2 = 1 + 1 = 2, c = sqrt(2) (irrational).
- (c) introduces the irrational hypotenuse — a key Unit 8 idea.

Problem 4 — Ladder problem

- h^2 + 6^2 = 10^2 -> h^2 = 64 -> h = 8 feet.
- Strong: draws and labels the triangle. Identifies HYPOTENUSE as the ladder (the longest side, opposite the right angle).
- Common error: assuming the 10 is a leg.

Problem 5 — Distance between points

- Horizontal leg: 7-1 = 6. Vertical leg: 10-2 = 8. Distance = sqrt(36+64) = sqrt(100) = 10.
- This is the DISTANCE FORMULA introduced via Pythagoras. Strong students DRAW the right triangle on the coordinate plane.
- Weak: tries to subtract coordinates directly.

Problem 6 — 3D box diagonal

- First diagonal of base: sqrt(3^2 + 4^2) = 5. Then box diagonal: sqrt(5^2 + 12^2) = sqrt(25+144) = sqrt(169) = 13 cm.
- Two applications of Pythagoras — this is the 3D extension.
- Strong: explicitly identifies the two right triangles. Weak: doesn't see two steps are needed.

Problem 7 — Rational vs irrational

- (a) sqrt(16) = 4 — rational (integer).

- (b) sqrt(2) — irrational (no fraction equals it; decimal is non-repeating, non-terminating).
- (c) 0.333... = 1/3 — rational.
- (d) pi — irrational.
- (e) 0.1010010001... — irrational (non-repeating, no pattern).
- Diagnostic: irrational = cannot be written as a/b where a, b are integers. Equivalently: non-terminating non-repeating decimal.

Problem 8 — Approximating sqrt(20)

- 4^2 = 16, 5^2 = 25. So 4 < sqrt(20) < 5.
- 4.4^2 = 19.36, 4.5^2 = 20.25. So 4.4 < sqrt(20) < 4.5.
- Without calculator! Strong students square integers and tenths to narrow it down.

Problem 9 — Pythagorean triples

- Both are Pythagorean triples. 5^2 + 12^2 = 25 + 144 = 169 = 13^2. 9^2 + 40^2 = 81 + 1600 = 1681 = 41^2.
- What's special: integer side lengths satisfying the theorem. The (3-4-5) family generalizes.
- Pythagorean triples connect arithmetic, geometry, and number theory.

Problem 10 — Han's claim

- Han is WRONG. Irrational numbers like pi = 3.14159... can be written as decimals but are NOT rational.
- Key distinction: rational numbers have TERMINATING or REPEATING decimal expansions. Irrationals are non-terminating AND non-repeating.
- Strong response gives a counterexample like pi or sqrt(2).

Problem 11 — Is 5-7-9 a right triangle?

- Check: 5^2 + 7^2 = 25 + 49 = 74. But 9^2 = 81. Since 74 ≠ 81, NOT a right triangle.
- Strong students recognize this is the CONVERSE of Pythagoras: if a^2 + b^2 = c^2, then the triangle is right.
- Strict: should compare a^2 + b^2 to c^2 where c is the LONGEST side.

Problem 12 — Why (sqrt(2))^2 = 2

- By DEFINITION: sqrt(2) is the (positive) number whose square is 2.
- Visually: a square with side sqrt(2) has area 2.
- Algebraically: (sqrt(x))^2 = x for x ≥ 0.
- Strong: cites the definition. This is the structural fact behind ALL the radical work.

Problem 13 — Mia and the converse

- Mia is RIGHT, and the technical name is THE CONVERSE OF THE PYTHAGOREAN THEOREM.
- Strong: distinguishes the theorem (right -> a^2+b^2=c^2) from its converse (a^2+b^2=c^2 -> right). Both are true here, but they say different things logically.
- Bonus: notes that this is a classic example where both a statement and its converse are true.

— end —