

Accelerated 7 • Unit 7 • Assessment Guide

Exponents and Scientific Notation

For the Parent or Teacher — What to look for

Unit 7 builds fluency with exponent properties — multiplying, dividing, powers of powers — and extends to zero and negative exponents. Scientific notation gives students a powerful tool for very large or very small numbers. Watch for **STRUCTURE** understanding (why the rules work), not just rote application.

Problem 1 — Basic exponent rules

- (a) 2^8 . (b) 5^8 . (c) 7^4 . (d) $2^4 * 3^4$ (or 1296 if computed).
- Diagnostic: do they MIX UP the rules? Common error: $2^5 * 2^3 = 2^{15}$ (multiplied exponents instead of added).
- Strong: writes the rule (add exponents when multiplying same base, etc.).

Problem 2 — Why $3^5 * 3^2 = 3^7$

- $3^5 = 3*3*3*3*3$. $3^2 = 3*3$. Multiplying: $(3*3*3*3*3)*(3*3) = 3*3*3*3*3*3*3 = 3^7$.
- This counts $5 + 2 = 7$ threes. THE conceptual basis for the product rule.
- Strong: writes this out as a chain of 3s; weak: just repeats the rule.

Problem 3 — Zero and negative exponents

- (a) $5^0 = 1$, $17^0 = 1$.
- (b) $2^{-3} = 1/8 = 0.125$. $10^{-2} = 1/100 = 0.01$.
- (c) $(1/3)^{-2} = 9$ (reciprocal squared).
- Critical errors to watch: $5^0 = 5$ or $5^0 = 0$; $2^{-3} = -8$ (treating exponent as multiplication).

Problem 4 — Diego's error (2^{-3})

- Diego thought the negative sign distributed to the result (negation). But negative exponent means RECIPROCAL: $2^{-3} = 1/2^3 = 1/8$.
- Strong response shows the PATTERN: $2^3=8$, $2^2=4$, $2^1=2$, $2^0=1$, $2^{-1}=1/2$, $2^{-2}=1/4$ -> each step divides by 2. So $2^{-3} = 1/8$.

Problem 5 — To scientific notation

- (a) $4.3 * 10^6$. (b) $6.7 * 10^{-4}$. (c) $5.972 * 10^{24}$.
- Format must be coefficient between 1 and 10 ($1 \leq |a| < 10$). Watch for $43 * 10^5$ or $0.43 * 10^7$ — both wrong forms.

Problem 6 — Scientific to standard

- (a) 320,000. (b) 0.000705.
- Counting decimal places carefully; common to be off by one (especially for negative exponents).

Problem 7 — Computation in scientific notation

- (a) $6 * 10^{10}$.
- (b) $2 * 10^6$.
- (c) $2 * 10^5 + 0.3 * 10^5 = 2.3 * 10^5$.
- (d) $5 * 10^4$.

- Diagnostic on (c): adding requires SAME power of 10. Many students try to just add the coefficients and exponents separately — wrong.

Problem 8 — Sunlight travel time

- $t = (1.5 \cdot 10^8) / (3 \cdot 10^5) = 0.5 \cdot 10^3 = 500$ seconds = 8.33 minutes $\approx$ 8 min 20 s.
- Strong: writes in scientific notation throughout and converts to minutes at the end.
- Weak: computes $150,000,000 / 300,000$ by hand — misses the point of the unit.

Problem 9 — Comparisons

- (a) $2^{10} = 1024 > 10^2 = 100$. Strong: notes $2^{10} \approx 10^3$.
- (b) $3.5 \cdot 10^{(-7)} > 9.2 \cdot 10^{(-8)}$ — because $10^{(-7)}$ is bigger than $10^{(-8)}$. (Common confusion: thinking $9.2 > 3.5$ means the second is bigger.)
- (c) $(2^5)^3 = 2^{15} = 32768$. $2^5 \cdot 2^3 = 2^8 = 256$. So $(2^5)^3$ is much bigger.
- Diagnostic on (b): does the student look at the EXPONENT first, then the coefficient?

Problem 10 — Why $5^0 = 1$

- Pattern: $5^3 = 125$, $5^2 = 25$, $5^1 = 5$, $5^0 = ?$ Each step divides by 5. So $5^0 = 5/5 = 1$.
- OR: $5^n / 5^n = 1$, but also $= 5^{(n-n)} = 5^0$. So $5^0 = 1$.
- Strong responses give one of these justifications. Weak: just states the rule.

Problem 11 — Bacteria doubling

- (a) After 20 min: 2. After 1 hr (60 min = 3 doublings): $2^3 = 8$. After 4 hrs (12 doublings): $2^{12} = 4096 \approx 4.1 \cdot 10^3$.
- (b) $N(t) = 2^{(3t)}$ where t is hours. (Three doublings per hour.)
- (c) After 24 hours: $2^{72} \approx 4.7 \cdot 10^{21}$. That's MUCH MORE than $3 \cdot 10^{13}$ — by 8 orders of magnitude.
- Exponential growth dramatically outpaces large numbers. A key Algebra 1 readiness insight.

Problem 12 — Mai's claim

- Verify $a=2$, $m=3$, $n=-5$: $2^3 \cdot 2^{(-5)} = 8 \cdot (1/32) = 1/4 = 2^{(-2)}$. And $m+n = 3+(-5) = -2$. So $2^{(-2)} = 1/4$.
- Verify $a=4$, $m=-2$, $n=-3$: $4^{(-2)} \cdot 4^{(-3)} = (1/16)(1/64) = 1/1024 = 4^{(-5)}$. $m+n = -5$.
- Mai is right. The exponent rules extend smoothly to negative exponents — a powerful structural insight.

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