Accelerated 7 · Unit 6 · Assessment Guide

Functions and Volume

For the Parent or Teacher — What to look for

Unit 6 introduces functions formally and applies them to volumes. The big idea: a function assigns ONE output to each input. Watch for whether students appreciate that linear functions are a special, simple kind of function — and that volume formulas are NON-linear functions of radius.

Problem 1 — Function or not?

- (a) NOT a function — multiple people share birthdays.
- (b) Function — each state has exactly one capital.
- (c) NOT a function — most positive numbers have TWO square roots (e.g., 9 -> 3 and -3).
- The discriminator: "one input, one output." Strong students can produce counterexamples (the same input giving two outputs).

Problem 2 — Apple function

- (a) f(n) = 0.75n.
- (b) f(8) = $6.00, the cost of 8 apples.
- (c) f(n) = 6 -> n = 8 apples. "How many apples cost $6?"
- Strong: distinguishes between EVALUATING the function (f(8)) and SOLVING (find n with f(n) = 6). These are different mental moves.

Problem 3 — Linear vs non-linear

- (a) Linear: y = 2x - 5.
- (b) Non-linear: y = x^2 (quadratic).
- (c) NON-linear: y doubles each step (3, 6, 12, 24); growth rate increases. Exponential pattern.
- (d) Linear (any straight line is linear).
- Key insight: linear means CONSTANT rate of change. Strong students check differences in tables; weak students assume y growing means linear.

Problem 4 — Distance vs time graph

- Shape: line going up (steady walk), horizontal segment (rest), then steeper line going DOWN (running back to start).
- Diagnostic: do they recognize that returning means distance DECREASES — the graph slopes down?
- Strong: labels axes (time on x, distance on y) and uses different slopes for walking vs running.

Problem 5 — Cylinder volume

- (a) V = pi*r^2*h = pi(9)(5) = 45pi m^3.
- (b) Doubling radius: V = pi(36)(5) = 180pi. Factor = 4 (because r is SQUARED).
- (c) Doubling height: V = pi(9)(10) = 90pi. Factor = 2 (height is linear).
- THE diagnostic: do they recognize that doubling radius QUADRUPLES volume, not doubles? This is the most important non-linear scaling insight in the unit.

Problem 6 — Cone vs cylinder

- (a) Cylinder: V = pi(16)(9) = 144pi cm^3.
- (b) Cone: V = (1/3)pi(16)(9) = 48pi cm^3.

- (c) Ratio = 1/3. The cone formula has the (1/3) factor because three cones of same base and height fill one cylinder.
- Strong: states the 1/3 relationship as a fact; even stronger students can describe a hands-on justification.

Problem 7 — Sphere volume scaling

- (a) V1 = (4/3)pi(125) = 500pi/3 cm^3. V2 = (4/3)pi(1000) = 4000pi/3 cm^3.
- (b) Factor of 8 (V2/V1 = 8). Because radius doubled, volume scales by 2^3 = 8.
- (c) General rule: multiplying radius by k multiplies volume by k^3.
- This generalizes to all 3D shapes: linear scaling by k -> volume scales by k^3. A key Algebra 1 readiness idea.

Problem 8 — Linear vs quadratic graphs

- y = mx + b is linear because the variable appears only to the first power; rate of change is constant.
- A = pi*r^2 is non-linear (quadratic) — A grows faster than r; rate of change INCREASES with r.
- Strong: explicitly connects "degree of the variable" to shape of graph.

Problem 9 — Cube doubling

- 2-cm cube: V = 8 cm^3. 4-cm cube: V = 64 cm^3.
- Volume increased by FACTOR 8 (2^3), not by factor 2.
- Sara is wrong — doubling each side multiplies volume by 2^3 = 8.
- This problem is a classic and CRITICAL. Many students think doubling = doubling. Without this insight, scaling problems in geometry and science fail.

Problem 10 — Paper rolled two ways

- Not necessarily the same volume! Consider an 8.5 x 11 sheet. Rolled with circumference 8.5: r ≈ 1.35, h = 11. V ≈ 63.3 cubic inches.
- Rolled with circumference 11: r ≈ 1.75, h = 8.5. V ≈ 81.7 cubic inches.
- Same SURFACE AREA (the paper) but DIFFERENT VOLUMES, because r^2 grows faster than h shrinks.
- Strong: uses the formula V = pi*r^2*h and observes that r is squared but h is linear.

Problem 11 — Cone filling at steady rate

- NOT linear. As water rises, the cross-sectional area increases (cone widens), so the same volume of water raises the water level by LESS.
- Graph: water height rises QUICKLY at first (narrow part), then SLOWS as it widens.
- Diagnostic: students who draw a straight line haven't connected geometry to function behavior.

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