Accelerated 7 • Unit 1 • Assessment Guide

Rigid Transformations and Congruence

For the Parent or Teacher — What to look for

Unit 1 introduces the formal language of rigid transformations and uses them to define congruence and prove facts about parallel lines and triangle angles. The key shift from grade 6/7 is precision of language and the move toward justification.

Problem 1 — Three transformations of triangle ABC

- (a) Translation: A(6,1), B(9,1), C(6,4). Watch for sign confusion on "down" (negative y).
- (b) Reflection across x-axis: A(1,-2), B(4,-2), C(1,-5). Common error: reflecting across y-axis instead.
- (c) Rotation 90° CCW about origin: (x,y) -> (-y,x). A(-2,1), B(-2,4), C(-5,1).
- Diagnostic: do they articulate the RULE for the rotation, or only get the answer right by tracing? Asking how they'd find the image of (a,b) tests deeper understanding.

Problem 2 — Composition of rigid motions

- Yes — a 180° rotation followed by a reflection is equivalent to a single reflection (a glide reflection or another reflection, depending on the axis).
- Strong: notes that compositions of rigid motions are themselves rigid motions; might explore specific case.
- Weak: insists on a unique "identity" for each move; doesn't see motions as composable functions.

Problem 3 — Preserved properties

- CD must also be 7 cm (distance is preserved). Any angles formed at A and B equal corresponding angles at C and D (angles are preserved).
- Justification should reference the DEFINITION of rigid motion as preserving distance and angle measure, not just say "because they're congruent."

Problem 4 — 180° rotation of parallel lines

- Key insight: a 180° rotation about ANY point not on a line takes the line to a parallel line.
- Strong: states this property, then explains that since m and n are parallel and on opposite sides of P, the rotation maps m to a parallel line through P's image — which must coincide with n.
- This problem is preparation for proving alternate interior angles are congruent.

Problem 5 — Same sides, different angles

- Not congruent. Same side lengths but different angles -> no rigid motion maps one to the other.
- Strong answer mentions that all sides matching is NOT enough — angles must also match.
- Weak: says they ARE congruent because all sides equal (a common rhombus/square confusion).

Problem 6 — Same area does not imply congruent

- Diego is wrong. Counterexample: 2 by 6 rectangle and 3 by 4 rectangle both have area 12 but are not congruent.
- Best counterexamples are simple and unambiguous. Watch for students who give only specific numbers without explaining why it disproves Diego.

Problem 7 — Angle relationships with parallel lines

- Eight angles total around two intersection points; given one (47°), the other angles are 47° or 133°.

● Strong: names relationships (vertical = same; corresponding = same; alternate interior = same; co-interior / supplementary = 180° - 47° = 133°).

● Weak: just writes 47° and 133° without identifying which is which.

Problem 8 — Triangle angle sum proof

● Angle F = 70°.

● Proof: draw line through one vertex parallel to opposite side. The three angles at that vertex form a straight line (180°). By alternate interior angles, two of those equal the other two angles of the triangle. So the triangle's three angles sum to 180°.

● This is one of the most important problems in the unit — first formal geometric proof. Look for: clear figure, named angles, references to alternate interior, conclusion.

Problem 9 — Determining triangles

● (a) 3-4-5: unique (right triangle, SSS).

● (b) 2, 3, 10: impossible (triangle inequality: 2+3 < 10).

● (c) 60-60-60: infinitely many (similar but not unique — could be any size).

● (d) AAS with included side specified: unique triangle.

● Diagnostic: (c) is the tricky one — students sometimes say "unique" because there's only one SHAPE.

Problem 10 — Inverses of transformations

● Both girls are partly right: translations are undone by translations, rotations by rotations, AND reflections by the SAME reflection (a reflection is its own inverse).

● Lin's instinct that reflections are "different" deserves credit — reflections reverse orientation, while translations and rotations preserve it.

● Strong response addresses orientation: rigid motions split into direct (translation, rotation) and indirect (reflection).

Problem 11 — Right triangles, legs 3 and 4

● Yes — congruent by SAS (or HL). The right angle is between the two legs, so 3, 90°, 4 fully determines the triangle.

● Foreshadows Pythagoras: hypotenuse must be 5, so once legs are fixed the third side is determined.

● Weak: says only one of the conditions is enough; doesn't recognize the included angle.

Problem 12 — Two sequences with same effect

● Many options. E.g., rotate 90° CCW about origin maps (2,3) -> (-3,2). OR reflect across y=x then reflect across x-axis. OR translate then reflect.

● Strong students recognize that two distinct sequences can produce the same MAPPING — the mapping is what matters geometrically. This is conceptually important for grade 8 / Algebra 1 (functions).

● Weak: claims any two different sequences are different motions even when they produce identical images.

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