Accelerated 6 · Unit 9 · Assessment Guide

Putting It All Together

For the Parent or Teacher — What to look for

Unit 9 weaves the year's threads together. Watch whether students can RECOGNIZE which tools to use in unfamiliar contexts. The deepest test: can they generalize beyond the specific numbers given?

Problem 1 — Lawn with circular flower bed (multi-step)

- (a) 96 m². (b) π·(1.5)² ≈ 7.065 m². (c) ≈ 88.935 m². (d) 88.935/2 ≈ 44.47 g. (e) 44.47 · $0.15 · 1.08 ≈ $7.20.
- ★ Strong: keeps multi-step work organized, with units throughout.
- Weak: loses track of unit conversions; computes tax on grams instead of price.

Problem 2 — Voting percentages and runoff

- Lin: 40%, Diego: 33⅓%, Mai: 26⅔%.
- No majority winner (largest is 40%).
- Runoff: Lin 12 + 4 = 16, Diego 10 + 4 = 14. Lin wins by 2.
- Strong: handles the runoff splitting cleanly.
- ★ Strong+: notes that in a 3-way race, a plurality winner may not be a majority winner — an important civic-math insight.

Problem 3 — Run course design

- (a) Flat 3000 m, Uphill 1250 m, Downhill 750 m.
- (b) Sum = 5000 ✓.
- (c) Flat: 750s; Uphill: 416.67s; Downhill: 125s. Total ≈ 1291.67s ≈ 21.5 min.
- (d) Average speed = 5000m/1291.67s ≈ 3.87 m/s.
- ★ KEY: average speed is NOT the simple average of 4, 3, 6 (which is 4.33).
- Strong: explains that average speed is WEIGHTED by TIME spent, not by DISTANCE. (Or equivalently: harmonic mean over equal-distance segments.)
- ★ This is a deep idea — the difference between arithmetic and weighted means.

Problem 4 — Water tank with pump and leak

- (a) Net = 12 − 2 = 10 L/min.
- (b) 480/10 = 48 min.
- (c) Pump on 480 min, so net 4800 L. But tank holds 480 — it FILLS in 48 min. After that, the pump can't add more. So at 4 p.m., it's still 480 L (FULL).
- Wait — does the pump turn off automatically? Or does excess overflow? Assume excess overflows: still full at 4 p.m.
- (d) Leak alone: 480 L / 2 L/min = 240 min = 4 hours.
- ★ Strong: addresses what happens when the tank reaches capacity.
- Weak: computes 'net L over 8 hours' = 4800 L without noticing it can't hold that much.

Problem 5 — City polling estimate

- (a) A: 62%, B: 38%.
- (b) A: 124,000, B: 76,000.

- (c) NO — sampling variability means each new sample gives slightly different numbers.
- (d) 5 friends is too small AND not random (friends are not a random sample of the city).
- ★ Strong: addresses both 'too small' and 'not random.'
- Weak: just says 'too few.' Misses the bias point.

Problem 6 — Same area: circle vs square perimeter

- Area = 100π → r = 10 → circumference = $20\pi \approx 62.83$ cm.
- Side of square = $\sqrt{(100\pi)} \approx 17.72$ → perimeter = ≈ 70.90 cm.
- ★ Circle has SMALLER perimeter.
- ★ Strong: states the GENERALIZATION — 'for a given area, the circle minimizes perimeter (isoperimetric inequality).'
- ★ Strong+: this is connected to why bubbles are spherical, drops are round, etc. Real geometric insight.

Problem 7 — Two trains meeting

- Train 1 at time t (hours after 9:00): 60t.
- Train 2 at time t (note: leaves at 9:30): $300 - 80(t - 0.5)$ for $t \geq 0.5$.
- Set equal: $60t = 300 - 80t + 40 = 340 - 80t$. So $140t = 340$ → $t = 340/140 = 17/7 \approx 2.43$ hours after 9:00, so about 11:26 a.m.
- Distance from A: $60 \cdot 17/7 \approx 145.7$ km.
- ★ Strong: handles the time offset correctly (train 2 starts 30 min later).
- Weak: ignores the 30-minute offset → gets the wrong intersection.

Problem 8 — 25% up then 25% down

- $200 → $250 → $187.50.
- Final is 93.75% of start, NOT 100%.
- ★ Reason: $(1.25)(0.75) = 0.9375$. The 25% decrease is taken from a LARGER number, so it subtracts more dollars than the 25% increase added.
- Strong: connects to Unit 6 problem on +50%/−50% and Unit 2 generalization $(1+p)(1-p) = 1 - p^2$.
- Strong+: notes that for p = 25%, the loss is 6.25% of original — exactly p^2.

Problem 9 — Single 30% vs 10%-then-20%

- 30% off: $\$80 \cdot 0.7 = \$56$.
- 10% then 20%: $\$80 \cdot 0.9 \cdot 0.8 = \$80 \cdot 0.72 = \$57.60$.
- Single 30% is BETTER by $1.60.
- ★ Generalization: $0.7 < 0.9 \cdot 0.8 = 0.72$. So 30% all at once is bigger savings than 10%+20% in stages.
- ★ Strong+: addresses why this is — '10%+20% on a smaller base means the second discount is on a smaller amount, so total savings is less than 30% of original.' Like compounding for losses.
- Weak: says they're the same because '10+20 = 30.' Classic additive misconception.

Problem 10 — Open-ended multi-strand problem

- Look for: at least 3 strands integrated (area, percent, ratio, equation, rational number, statistics).
- Strong: the strands genuinely interact (not just 3 separate parts).
- Strong+: scenario is realistic and the math feels necessary (not contrived).
- Weak: 3 strands appear but are independently computable; or one strand dominates.

— end —